# Variation of Implied Volatility and Return Predictability<sup>\*</sup>

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#### Abstract

The standard deviations of implied volatility levels, implied volatility innovations, and of the volatility premium all have significant predictive power for underlying returns. This return predictability is not explained by that previously documented for the levels of these three variables, or by firm characteristics and common risk factor models. We find support for interpreting the standard deviations of these option-based measures as forward-looking proxies of heterogeneous beliefs. The negative relationship between our three measures and future underlying returns we observe is consistent with the Miller (1977) theoretical result that divergence of investor opinions leads to lower expected returns.

Keywords: Options; Implied Volatility; Return Predictability; Belief Heterogeneity

JEL classification: G12, G13, G14

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# 1 Introduction

Option prices and implied volatilities reflect investor expectations about the underlying asset. We find that the standard deviations of known measures of these expectations predict stock returns, controlling for the levels of the measures themselves, as well as risk factors and firm characteristics. Furthermore, we show that the standard deviations of these measures of investor beliefs correspond to market uncertainty and forward-looking belief heterogeneity.

Theoretical results (Chowdhry and Nanda, 1991; Back, 1993; Easley, O'Hara, and Srinivas, 1998) and a wealth of empirical evidence (see, e.g., Chakravarty, Gulen, and Mayhew, 2004; Ni, Pan, and Poteshman, 2008; Bali and Hovakimian, 2009; Conrad, Dittmar, and Ghysels, 2013; An, Ang, Bali, and Cakici, 2014) suggest the option market contains unique information about the underlying assets. The empirical findings on return predictability from implied volatility data focus on innovations, levels, and spreads of implied volatility as measures of investor beliefs about the future. The standard deviations of these known predictive measures by definition represent the variability in investor beliefs they reflect. The implications of investor belief variability for the underlying asset's future performance have thus far not been studied. The aim of this study is to explore this issue.

In particular, the volatility premium (Bali and Hovakimian, 2009) and implied volatility innovations (An, Ang, Bali, and Cakici, 2014) predict future underlying returns, and therefore must by definition reflect investor beliefs about them. Implied volatility itself reflects investor beliefs about the overall risk of the underlying (Patell and Wolfson, 1979, 1981; Poterba and Summers, 1986; Whaley, 2000). We conjecture that the standard deviations of these three measures of investor beliefs increase with the variability or heterogeneity of these beliefs, and find evidence consistent with this view. Our three measures of the heterogeneity of investor beliefs impounded into the forward-looking options market enable us to more accurately address the relationship between belief heterogeneity and future returns than prior contemporaneous or backward-looking measures have done. We explore the relation between our forward-looking measures of heterogeneous beliefs, as well as ones from prior work, and future stock returns. Our forward-looking measures of belief heterogeneity help address an existing debate about the effects of belief heterogeneity on future stock returns: the Miller (1977) overvaluation theory predicts a negative relation between investor belief differences and stock returns, while the risk theory proposed by Williams (1977) predicts a positive one. Miller (1977) states that since divergence of opinion is likely to increase with risk, expected returns will be lower for risky securities as their prices will have been bid up by an overly optimistic minority. Contrary to this, Williams (1977) introduces heterogeneous beliefs into the Capital Asset Pricing Model and finds that the regression relationship between excess returns on any security and the associated beta has a non-zero intercept, consistent with higher expected returns.

There is a similar debate in empirical studies on this topic. Diether, Malloy and Scherbina (2002) find higher dispersion in analysts earnings forecasts leads to lower future returns than otherwise similar stocks. They interpret dispersion in analysts forecasts as a proxy for heterogeneous beliefs, finding empirical evidence consistent with Miller (1997). Andersen, Ghysels and Juergens (2005) focus on the pricing of uncertainty, rather than risk, measured as the degree of disagreement on macroeconomic and financial variables from the Survey of Professional Forecasters. They find empirical evidence for an uncertainty premium, consistent with Williams (1977). We add to this literature by documenting a correlation in option-based return predictor variabilities with existing heterogeneous belief proxies, which along with the strongly negative relationship between these variabilities and future returns supports the theory of Miller (1977) rather than Williams (1977): belief heterogeneity results in lower future returns. Portfolio sorts on standard deviations of our three implied volatility measures deliver monthly abnormal returns ranging from -.56% to -1.00%. Our paper contributes to the literature that examines the connection between implied volatility measures from the options market and the stock market at the individual firm level. It also contributes to the literature on the price impact of heterogeneous beliefs.

The intuition behind this empirical investigation is illustrated by the volatility premium, the spread between implied and historical volatility. In the option pricing literature two explanations have been advanced for its existence: expected volatility risk of the underlying (Bali and Hovakimian, 2009), and behavioral overreaction to realized gains and losses in the underlying (Goyal and Saretto, 2009). While remaining agnostic about the relative importance of these potential explanations, we draw on the common fact that a positive volatility premium indicates increased concerns in the market about future volatility risk. The standard deviation of this volatility risk measure, intuitively, should be a proxy of heterogeneous beliefs: if investors are homogeneous in their assessment of significant (insignificant) volatility risk, the volatility premium will be large (small) and consistent. Its standard deviation will therefore be low, and the opposite will obtain if investor beliefs alternate. The standard deviation of the volatility premium as the spread between atthe-money (ATM) put option implied volatility and historical volatility,  $\sigma_{I/H,P}$ , is our first measure of heterogeneity of beliefs about the underlying asset.

We extend this intuition for examining the standard deviation of predictive variables as a proxy of heterogeneous beliefs to two other option-based measures of investor expectations about the underlying. The first is the innovation in ATM put implied volatilities. This measure is shown to reflect investor expectations by An, Ang, Bali, and Cakici (2014): large past innovations in put option implied volatilities predict lower stock returns. Therefore the standard deviation of this measure of investor beliefs,  $\sigma_{\Delta IV,P}$ , is a measure of belief heterogeneity about the expected returns to the underlying stock.<sup>1</sup>

The second is the level of implied volatility itself. Prior studies show that the level of implied volatility will be higher (lower) for firms that are perceived to be more (less) risky (Patell and Wolfson, 1979, 1981; Poterba and Summers, 1986; Whaley, 2000). The standard deviation of the implied volatility,  $\sigma_{IV,P}$  is thus a measure of heterogeneous beliefs about

<sup>&</sup>lt;sup>1</sup>For the sake of brevity we omit a similar analysis for innovations in ATM call option implied volatilities. While the call volatility innovation predicts returns in the opposite direction, its standard deviation reflects a similar heterogeneity in beliefs.

overall firm risk.

Using these three measures of variability in investor beliefs, we create portfolio sorts and also examine their cross-sectional price impact for 4,911 stocks from January 1996 to August 2015. First we form monthly portfolios on quintiles of  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$ . This sorting procedure results in 5 portfolios per standard deviation, whose value weighted and equally weighted performance we track over the subsequent calendar month. To account for potential explanations such as firm characteristics or the previously documented return predictability of the level of the same implied volatility measures whose standard deviations we now consider, we next create double sorts on size, book to market, and means of the predictive measures with  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$ . Both univariate sorts on the  $\sigma$ s as well as double sorts on  $\sigma$ s controlling for the levels of the predictive variables, size, and book to market all show a negative relation to future stock returns. Next we use crosssectional regressions to confirm that  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$  are priced in the cross-section of stock returns, controlling for the level of the measures, size, book to market, historical and idiosyncratic volatility, and stock and option liquidity.

We next turn to potential causes of IVF variability. We run Fama-MacBeth crosssectional regressions of  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$  on other proxies of heterogeneous beliefs, controlling for the means of these IV measures. We find that other proxies of heterogeneous beliefs are significantly related to the standard deviations of option-based return predictors. Our results are consistent with Miller's (1977) theorized negative relationship between heterogeneous beliefs and expected returns.

The Miller (1977) theoretical findings about the negative relationship between investor belief heterogeneity and expected returns are derived under the assumption of short sale constraints. We test the importance of this assumption to our results using a natural experiment in short sale constraint reduction, the SEC Regulation SHO, following the approach of Fang, Huang, and Karpoff (2016). This SEC regulation selected a third of the Russell 3000 constituents at random as pilot stocks for exemption from short-sale price tests during 2005-2007. This exogenous shock to short sale constraints allows us to apply difference-in-difference tests to the determinants of our  $\sigma$  belief heterogeneity measures, as well as to their predictive power for future returns. We find some evidence that  $\sigma$  measures increase for the pilot stocks, but we find no effect on return predictability.

The rest of this paper is organized as follows. In Section II, we describe the data and variable construction. Section III tests the relationship between our standard deviations of implied volatility measures and future stock returns. In Section IV we confirm the relation between the  $\sigma$  measures and other proxies of heterogeneous beliefs and find their determinants. Section V uses the SEC 2005-2007 Reg SHO pilot program as a natural experiment to test whether short-sale constraints affect the negative relationship we document between  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ ,  $\sigma_{IV,P}$  and future stock returns. Section VI concludes.

#### 2 Data and Variable Construction

In this section we describe the data and the methods used to compute our IVF variability measures. Our data on option implied volatilities comes from the option price file in IvyDB's OptionMetrics. We begin with daily option implied volatility data for all puts for all stocks from January 1996 to August 2015. We exclude options with a missing bid price or ask price, a bid price less than or equal to 0, an ask price less than or equal to the bid price, and a bid-ask spread less than the minimum spread (\$0.05 for options with prices less than \$3.00, and \$0.10 for options with prices greater than or equal to \$3.00). To ensure the options have enough liquidity, we only include put options with time to expiration of between 10 and 60 days, and eliminate options with zero open interest and volume. We also eliminate options where the special settlement flag in the OptionMetrics database is set, and options with missing implied volatilities and deltas. Finally, we eliminate options that violate arbitrage conditions. For put options, we require that the bid price be less than the strike and that the ask price be at least as large as the difference between the strike price and the spot price. We also collect liquidity data on the volume and open interest of ATM put options.

Additionally, we obtain data on stock returns from CRSP, calculating monthly returns from 1996 to 2015 for all individual securities with common shares outstanding. We eliminate utilities (SIC codes between 4900 and 4999) and financial companies (SIC codes between 6000 and 6999) from our sample. We obtain the data from Compustat to compute book-tomarket ratios. The final sample consists of 238,847 firm-month combinations (4,911 firms) from January 1996 to August 2015.

Our first implied volatility measure is the difference between daily average of implied volatility of ATM puts and the realized volatility over the previous year, a modified version of Bali and Hovakimian (2009) and Goyal and Saretto (2009). Goyal and Saretto (2009) find that firms that experience losses subsequently have a larger gap between implied and historical volatilities, consistent with an overstated perception of riskiness hypothesized by Barberis and Huang (2001). According to that hypothesis, investor risk perceptions are asymmetric in gains and losses, with losses increasing risk perception and gains reducing it. On the other hand, Bali and Hovakimian (2009) find a significant relation between underlying returns and the realized-implied volatility spread, indicative of informed trading in options by investors with private information. We define at-the-money (ATM) puts for firm *i* on day *t* as put options with delta between -0.625 and -0.375 with time to expiration between 10 and 60 days and denote their daily average implied volatility as  $\overline{IV}_{i,t}^{ATM(P)}$ .<sup>2</sup> We estimate realized volatility,  $RV_{i,t}$ , as the standard deviation of daily returns over the prior 365 days.

$$I/H, P_{i,t} = \overline{IV}_{i,t}^{ATM(P)} - RV_{i,t}$$
(1)

We compute the monthly average and standard deviation of I/H,  $P_{i,t}$  and denote them as

 $<sup>^{2}</sup>$ As a robustness check, we also use options with time to expiration between 30 to 91 days, between 10 to 91 days, and all available maturities. Our results remain the same. We also consider an open interest-weighted daily average, as well as alternative definition of moneyness, defining ATM puts as the ratio of strike to spot between 0.95 and 1.05 or ATM puts as the ratio of strike to spot between 0.975 and 1.025. Finally, we also obtain the same results when we calculate our spreads from the OptionMetrics volatility surface dataset instead of traded options data.

 $\mu_{I/H,P}$  and  $\sigma_{I/H,P}$ , the former of which controls for the documented return predictability (Bali and Hovakimian, 2009) while the latter of which is our first measure of investor belief heterogeneity.

Following An, Ang, Bali and Cakici (2014), our second implied volatility measure is the innovation in implied volatilities of put options. As documented by the authors, increases in put option implied volatilities predict decreases in next month's stock returns. We calculate the innovation in implied volatility of put options for firm i on day t as the daily change in daily average ATM put implied volatilities. That is,

$$\Delta IV, P_{i,t} = \overline{IV}_{i,t}^{ATM(P)} - \overline{IV}_{i,t-1}^{ATM(P)}$$
(2)

We compute the daily average of ATM put implied volatility for each firm, and take the first difference. Then we calculate the monthly average  $\mu_{\Delta IV,P}$  and the monthly standard deviation  $\sigma_{\Delta IV,P}$  of  $\Delta IV, P$ . The former controls for the established price impact of IV innovations (An, Ang, Bali, and Cakici, 2014), while the latter represents our second new measure of variability of the shape IVF.

Our final measure is the daily average of ATM put implied volatilities:

$$IV, P_{i,t} = \overline{IV}_{i,t}^{ATM(P)} \tag{3}$$

We compute the monthly mean  $\mu_{IV,P}$  and standard deviation  $\sigma_{IV,P}$  from the daily ATM puts implied volatilities. The former controls for the overall market beliefs about firm risk (Patell and Wolfson, 1979, 1981; Poterba and Summers, 1986), while the latter is our final measure of the heterogeneity of beliefs regarding it. The more volatile the three measures, the higher our expected divergence of investor opinion about the underlying asset.

In Table I, we present descriptive statistics for the standard deviations and means of our three measures, corresponding volume and open interest, as well as firm-specific characteristics: market capitalization, book-to-market ratio and realized volatility over the past year. We report means, medians, and standard deviations as well as 5th and 95th percentiles across securities during the sample period.

We include data on ATM put option volume and open interest to control for asset liquidity and price pressure issues, and realized volatility to control for the baseline level of risk in the firm. We obtain quarterly book value of equity of the firm from COMPUSTAT, and market value of equity from CRSP.<sup>3</sup> We also include controls for *LEVERAGE* as the ratio of longterm debt to total assets, *BASPREAD* as the monthly average bid-ask spread, *BETA* as the market coefficient from the Fama and French (1993) three factors, Carhart (1997) momentum factor plus Pastor and Stambaugh (2003) liquidity factor five-factor model,<sup>4</sup> *DISPERSION* as the IBES dispersion of analyst forecasts, and *IDIOVOL* as the idiosyncratic volatility measured by standard deviation of five-factor model residuals.<sup>5</sup>

We present the cross sectional medians of the  $\sigma$  measures across all stocks in our sample for each month during the sample period in Figure 1 to highlight their variation. Consistent with a heterogeneous beliefs interpretation, the level of variability in the  $\sigma$ s is higher during the dot-com and financial crises (in dashed grey) as periods of high uncertainty, and lower during the preceding expansions (shaded grey) as period of low uncertainty.

The  $\sigma_{I/H,P}$  and  $\sigma_{IV,P}$  measures peak in 2001 during the height of the Internet bubble, decline through 2007, then peak again in 2009 during the financial crisis. The  $\sigma_{\Delta IV,P}$ measure has less variability between years, but varies significantly within each year. The results in Figure 1 imply that the standard deviations of implied volatility measures are related to the macroeconomic environment in ways consistent with a heterogeneous beliefs interpretation. Further supporting this view, the three standard deviations of IV measures are highly correlated with each other consistent with a common signal.

 $<sup>^{3}</sup>$ Alternative results for annual book value to account for less missing values in annual Compustat produces similar results.

<sup>&</sup>lt;sup>4</sup>We also consider BETA estimates from the CAPM and the Fama and French (1993) three factor model, and the results are similar.

<sup>&</sup>lt;sup>5</sup>Idiosyncratic volatility estimates from the CAPM and the Fama and French (1993) three factor model are again consistent with reported findings.

Table II shows the cross-sectional correlations between the variables. The lower triangular of the correlation matrix reports Pearson correlations between each variables while the upper triangular matrix presents the non-parametric Spearman correlation matrix. We report insignificant coefficients in italics. The correlations between our standard deviations of the three  $\sigma$  measures range from 0.408 to 0.957, which show that our measures are highly correlated with each other. The  $\mu_{IV,P}$  are highly correlated to its corresponding standard deviation  $\sigma_{IV,P}$  at 0.588 (0.575 for Pearson), the correlation between the mean and standard deviation of  $\Delta IV, P$  spread is -0.066 (-0.072 for Pearson) and the correlation between the mean and standard deviation of I/H spread is 0.125 (0.256 for Pearson). The correlations between standard deviations and average levels of IV measures imply that controlling for the average levels will be important in establishing additional return predictability for the  $\sigma$ measures net of the level  $\mu$ s. Standard deviations of our three measures are negatively correlated with market capitalization and with the book-to-market ratio. We therefore include these variables as additional controls in our return predictability tests.

We next consider whether  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$  behave like mean-reverting (stationary) or random-walk (nonstationary) processes. To test this we perform the augmented Dickey-Fuller tests for all firms. The results are reported in Table III as the percentage of firm-level time series for which we can reject the null hypothesis of a unit root (nonstationary) process at the 1%, 5% and 10% levels for both  $\sigma$ s and  $\mu$ s of a given underlying's implied volatility measures. The results show that 76.47% to 76.84% time series of  $\sigma$  are significant at the 1% level, rejecting the null hypothesis that  $\sigma_t$  is non-stationary and 85.23% to 85.95% time series of  $\sigma$  are indeed stationary. Meanwhile, 79.24% to 83.55% time series of  $\mu$  are significant at the 1% level and 87.19% to 89.39% time series of  $\mu$  are stationary.

### 3 Variation of Implied Volatility and Return Predictability

We now examine whether the standard deviations of the volatility premium, implied volatility innovation, and level of implied volatility have predictive power for underlying returns. If these  $\sigma$  measures represent variability in investor beliefs regarding the underlying asset we expect to find a significant relationship. A positive relationship would be consistent with the Williams (1997) risk theory, while a negative one would be consistent with the Miller (1997) overvaluation theory.

# 3.1 Portfolio Sorts

We begin our analysis with monthly quintile portfolio sorts. Each month, we rank each firm on the basis of  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$ . We then use the ranking to form both equaland value-weighted portfolios of the underlying firms over the subsequent calendar month, holding the standard deviation rank fixed. This gives us 15 equal-weighted and 15 valueweighted stock portfolios with returns sampled at the monthly frequency over the period January 1996 through August 2015.

In Table IV Panel A we present results for the 15 equal-weighted portfolios sorted by magnitude of standard deviation of each of the three IV measures. The table reports excess returns along with abnormal performance relative to standard benchmarks. We benchmark performance using the Capital Asset Pricing Model (CAPM), Fama and French (1993) three-factor model (FF3), Fama and French (1993) three factors plus the Carhart (1997) momentum factor four-factor model (FFC4) and Fama and French (1993) three factors, the Carhart (1997) momentum factor plus the Pastor and Stambaugh (2003) liquidity factor five-factor model (FFCP5). We follow the standard procedure of forming zero-cost portfolios long the stocks in the highest quintile of IV measure volatility and short the stocks in the lowest quintile. To control for the autocorrelation in returns the t-statistics are adjusted using Newey and West (1987) standard errors with a lag of 6 months. The equal-weighted portfolio returns in Panel A demonstrate a negative relation between standard deviations of our three IV measures and future stock returns over the subsequent month. The excess returns, as well as the CAPM, FF3, FFC4 and FFCP5 alphas of the Q5-Q1 zero-cost portfolio are significantly negative.  $\sigma_{\Delta IV,P}$ , the standard deviation of the IV innovation, has significantly negative monthly abnormal returns relative to all benchmark models ranging from -0.68% at the 1% significance level relative to the FFC4 model to -0.87% significant at the 1% level relative to the CAPM model. The raw excess returns on  $\sigma_{\Delta IV,P}$ are negative and significant at the 10% level. We observe similar performance for  $\sigma_{I/H,P}$ , the standard deviation of the spread between implied volatility of at-the-money puts and realized volatility: the abnormal negative returns over the subsequent month range between -0.66% and significant at the 1% level relative to the FFC4 model and -0.96% significant at the 1% level for the CAPM model. The portfolio results for  $\sigma_{IV,P}$  are also similar. These results provide preliminary evidence for our first novel contribution: there is a statistically significant negative relation between standard deviations of option-based measures of investor beliefs and future stock returns.

Table IV Panel B presents analogous results for 15 value-weighted portfolios sorted on the standard deviations of our option-based investor expectations measures. This alternative weighting method de-emphasizes the role of small stocks in portfolio abnormal returns observed in Panel A. As before,  $\sigma_{I/H,P}$  has significantly negative abnormal returns relative to all benchmark models ranging from -0.72% at the 1% significance level relative to FF3 model, to -1.00% at the 1% significance level relative to the FFCP5 model. The raw excess returns on  $\sigma_{I/H,P}$  are negative but insignificant.  $\sigma_{\Delta IV,P}$  has significantly negative monthly abnormal returns relative to all benchmark models, too. However,  $\sigma_{IV,P}$  has relative weaker results, though it still has significantly negative abnormal returns relative to all benchmark models ranging from -0.56% at the 5% significance level relative to the FFC4 model to -0.87% at the 5% significance level relative to the CAPM model. The raw excess returns on  $\sigma_{IV,P}$ are negative, and again insignificant. Overall, this suggests that our results are not driven by the small firms in the sample.

In Table V we report mean and median firm characteristics of quintile portfolios formed on the standard deviations of the three implied volatility measures. Specifically, we summarize the monthly average level of the measure,  $\mu$ s, the market value of the firm's stock, MV, the book to market ratio, BM, and liquidity measures of the stock and option markets on the firm. We report the monthly share trade volume, VOLUME, and monthly average of daily ATM put volume and open interest,  $VOL_{P,ATM}$  and  $OI_{P,ATM}$  respectively. The mean and median of the monthly average of the implied volatility measures is monotonically increasing in standard deviation of IV, P and decreasing in standard deviation of  $\Delta IV, P$ . This relationship between the means and standard deviations of option-based investor belief measures implies that both the first and second moments need to be taken into account in a return predictability context, especially since the first moments have already been shown to have predictive power in the literature (Goyal and Saretto, 2009; Bali and Hovakimian, 2009; An, Ang, Bali and Cakici, 2014). However, the mean and median of I/H, P is nonmonotonically related with  $\sigma_{I/H,P}$ . Notably, the mean and median market capitalization MV are monotonically decreasing in standard deviations of three IV measures. The mean and median book-to-market ratios BM are also monotonically decreasing for  $\sigma$  portfolios as well, while all three liquidity measures are highest for the median portfolio.

#### **3.2** Double Sorts

As demonstrated in Table II, the standard deviations of IV measures are correlated with market capitalization. To test whether  $\sigma$  has predictive power for returns in excess of firm size we use a double sort procedure on  $\sigma$  and size (Ang, Hodrick, Xing, and Zhang, 2006; 2008; Boyer and Vorkink, 2014). We begin by sorting firms into size deciles and then into two portfolios by  $\sigma$  within each size decile. We then average the one-period return across all deciles to create returns of two equally weighted portfolios with similar levels of size but different  $\sigma$ . Then we reverse this procedure, and first sort firms into  $\sigma$  deciles and then into two size portfolios within each  $\sigma$  decile. Averaging the one-period returns across all  $\sigma$  portfolios, we create returns of two portfolios with similar levels of  $\sigma$  but different size.

We compare resulting portfolio abnormal returns, testing whether the predictive power we observe for standard deviations of our implied volatility measures depends on firm size. As in Table IV, we measure abnormal returns using the CAPM, FF3, FFC4, and FFCP5 models. We report differences in raw returns as well as abnormal returns across the two conditionally sorted portfolios along with Newey-West (1987) t-statistics controlling for 6 lags' autocorrelation in Table VI. We report results controlling for size in left panel of Table VI. For  $\sigma_{I/H,P}$  the abnormal returns spread ranges from -0.32% at the 1% significance level relative to the FFC4 model to -0.47% significant at the 1% level relative to the CAPM model. For  $\sigma_{\Delta IV,P}$  the abnormal returns spread ranges from -0.49% at the 1% significance level relative to the FF3 model to -0.49% significant at the 1% level relative to the CAPM model. The spread between raw excess returns of the two conditional controlled portfolios is also negative and significant at the 1% level. The results for  $\sigma_{IV,P}$  are similar: the abnormal returns spread ranges from -0.30% at the 1% significance level relative to the FFC4 model to -0.45% significant at the 1% level relative to the CAPM model. After controlling for size, we find the a persistent negative relationship between abnormal return and  $\sigma$ . These findings indicate that our return predictability results for  $\sigma s$  are not explained by variation in firm size.

We report results for size sorts controlling for  $\sigma$ s in right panel of Table VI. For  $\sigma_{I/H,P}$  the abnormal returns spread has no statistical significance. For  $\sigma_{\Delta IV,P}$  the abnormal returns spread ranges from an insignificant -0.12% to -0.26% with significance at the 10% level relative to the FFCP5 liquidity model. The results for  $\sigma_{IV,P}$  are also insignificant. After controlling for  $\sigma$ s, the predictive power of size for stock returns virtually disappears. This further indicates that the negative relationship between  $\sigma$  and returns is not driven by the size effect.

Table II also shows that  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$  are correlated with book-to-market

ratios. We use the same double sort procedure to test the predictive power of  $\sigma$  net of BM. After creating two portfolios with similar BM but different  $\sigma$ , as well as similar  $\sigma$  but different BM, we compare their abnormal returns in Table VII.

Controlling for BM in the left panel of Table VII, for  $\sigma_{I/H,P}$  the abnormal returns spread ranges from -0.39% at the 1% significance level relative to the FFC4 model to -0.57%significant at the 1% level relative to the CAPM model. For  $\sigma_{\Delta IV,P}$  the abnormal return spread ranges from -0.42% at the 1% significance level relative to the FF3 model to -0.56% significant at the 1% level relative to the CAPM model. The results for  $\sigma_{IV,P}$  are similar again: the abnormal returns spread ranges from -0.38% at the 1% significance level relative to the FFC4 model to -0.58% significant at the 1% level relative to the CAPM model. The excess return spreads are also negative and significant at 10% level for all  $\sigma$ s. After controlling for book-to-market ratio, we find the spreads in abnormal returns from different asset pricing models are large and significant at the 1% significance level. These findings support that our results are not entirely subsumed by the book-to-market effect.

Controlling for  $\sigma$ s in the right panel of Table VII, the abnormal returns spread for  $\sigma_{I/H,P}$ ranges from -0.01% to 0.27% with weak significance for the FFC4 and FFCP5 models at the 10% level. For  $\sigma_{\Delta IV,P}$  the abnormal returns spread ranges from an insignificant 0.00% to a weakly significant 0.29% relative to the FFCP5 model. The results for  $\sigma_{IV,P}$  are similar, as only alphas relative to the FFC4 and FFCP5 models are significant at the 10% level. These results further support that the negative relationship between  $\sigma$  and future returns is not driven by the value effect.

We also test whether the predictive power of  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$  is due to the levels of the option-based investor belief measures from which they originate: the I/H volatility premium (Bali and Hovakimian, 2009), the  $\Delta IV$  volatility innovation (An, Ang, Bali, and Cakici, 2014), and the IV implied volatility. We follow the same double sort procedure: first we sort firms by the monthly average level of each measure  $\mu$  into deciles and within each  $\mu$  decile we independently sort firms into two portfolios by corresponding  $\sigma$ , the standard deviation of the measures. The result is two portfolios with similar levels  $\mu$  of investor belief measures, but different variability  $\sigma$ . We then reverse this procedure, and first sort firms by  $\sigma$  into deciles and then within each  $\sigma$  decile sort firms by  $\mu$ s into two bins, producing portfolios with similar levels of variability in investor belief measures  $\sigma$  but different levels of these beliefs  $\mu$ . We compare their abnormal returns to see whether the predictive power we observe for the standard deviations of option-based measures of investor expectations depends on the level  $\mu$  of investor expectations, and vice versa.

We report results of  $\sigma$  sorts controlling for  $\mu$ s in left panel of Table VIII. For  $\sigma_{I/H,P}$  the abnormal returns spread ranges from -0.29% at the 1% significance level relative to the FFC4 model to -0.42% significant at the 1% level relative to the CAPM model. The  $\sigma_{\Delta IV,P}$  abnormal return spread ranges from -0.21% significant at the 5% level relative to the FFC4 model to -0.31% significant at the 5% level relative to the CAPM model. Finally, the  $\sigma_{IV,P}$  abnormal returns range from -0.15% at the 5% significance level relative to the FFC4 model to -0.17% significant at the 1% level relative to the CAPM model. That is, after controlling for the levels  $\mu$  of previously documented investor belief measures we still find significant abnormal returns from sorts on their standard deviation  $\sigma$ s. This indicates that the predictability of the standard deviations of IV measures is in excess of  $\mu$ s.

The right panel of Table VIII presents analogous results for  $\mu$  sorts controlling for  $\sigma$ s. For  $\sigma_{I/H,P}$  and  $\sigma_{\Delta IV,P}$  the abnormal return spreads are insignificant for all benchmark models. The results for  $\sigma_{IV,P}$  are only marginally stronger with the CAPM and FF3 models finding abnormal returns significant at 10% level. These results support that the negative relationship between standard deviations of option-based investor beliefs and future returns is not driven by the their corresponding levels  $\mu$ s. Furthermore, the observation that controlling for  $\sigma$ s eliminates return predictability for the levels  $\mu$ s documented in prior literature (Bali and Hovakimian, 2009; An, Ang, Bali, and Cakici, 2014) invites additional research into the importance of the variability of known option-based investor belief measures.

Finally since our excess return of Q5-Q1 in IX is insignificant or weakly significant for

our three measures, one concern is that we may capture a betting-against-beta effect. Hence we test whether the predictive power of  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$  is subsumed by that of beta. We again follow the same double sort procedure: first we sort firms by beta into deciles and within each beta decile we independently sort firms into two portfolios by corresponding  $\sigma$ , the standard deviation of the measures. The result is two portfolios with similar levels of beta, but different variability  $\sigma$ . We then reverse this procedure, and first sort firms by  $\sigma$  into deciles and then within each  $\sigma$  decile sort firms by beta into two bins, producing portfolios with similar levels of variability in investor belief measures  $\sigma$  but different levels of these beliefs beta. We compare their abnormal returns to see whether the predictive power we observe for the standard deviations of option-based measures of investor expectations depends on beta, and vice versa.

We report results of  $\sigma$  sorts controlling for beta in left panel of Table IX. For  $\sigma_{I/H,P}$  the abnormal returns spread ranges from -0.29% at the 1% significance level relative to the FFC4 model to -0.45% significant at the 1% level relative to the CAPM model. The  $\sigma_{\Delta IV,P}$  abnormal return spread ranges from -0.38% significant at the 1% level relative to the FFC4 model to -0.49% significant at the 1% level relative to the CAPM model. Finally, the  $\sigma_{IV,P}$  abnormal returns range from -0.28% at the 1% significance level relative to the FFC4 model to -0.45% significant at the 1% level relative to the CAPM model. Finally, the  $\sigma_{IV,P}$  abnormal returns range from -0.28% at the 1% significance level relative to the FFC4 model to -0.45% significant at the 1% level relative to the CAPM model. That is, after controlling for beta we still find significant abnormal returns from sorts on their standard deviation  $\sigma$ s. This indicates that the predictability of the standard deviations of IV measures is in excess of beta.

The right panel of Table IX presents analogous results for beta sorts controlling for  $\sigma$ s. For all our measures the abnormal returns spreads are insignificant for all benchmark models. These results support that the negative relationship between standard deviations of option-based investor beliefs and future returns is not driven by a betting-against-beta effect.

#### **3.3** Cross-sectional regressions

The double sorts in the prior section support the existence of predictive power of  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$  in excess of that from the levels of I/H,  $\Delta IV$ , and IV, size, and book to market ratio individually. However, it is still possible that predictive signal in  $\sigma$  is explained by a simultaneous combination of multiple characteristics. We apply a monthly cross-sectional regression approach to test the predictability of excess stock returns from our  $\sigma$  measures controlling for a set of predictive variables:

$$r_{i,t+1} = a_0 + a_1 \sigma_{i,t} + a_2 \mu_{i,t} + a_3 M V_{i,t} + a_4 B M_{i,t} + a_5 R V_{i,t} + a_6 V O L_{i,t} + a_7 O I_{i,t} + a_8 V O L U M E_{i,t} + a_9 I D I O V O L_{i,t} + \epsilon i, t$$
(4)

We control for  $\mu$ s, the monthly average levels of I/H,  $\Delta IV$ , and IV, MV, the logtransformed market value of the firm, BM, the book-to-market ratio, RV, the realized volatility of returns over the prior year, and liquidity measured by monthly averages of daily ATM put option volume, VOL, and open interest, OI, as well as the monthly stock volume VOLUME. To test or channel of  $\sigma$  return predictability via a superior forwardlooking measure of investor belief heterogeneity, we also control for IDIOVOL, the monthly idiosyncratic volatility over the prior 60 months. This variable is a backward-looking proxy for heterogeneous beliefs (Friesen, Zhang, and Zorn, 2012), and allows us to isolate the novel component of belief heterogeneity from forward-looking options data. Table X presents the cross-sectional regression results of excess returns on the standard deviations of implied volatility measures with controls.

Consistent with prior results, the standard deviation of the I/H spread,  $\sigma_{I/H,P}$ , has a coefficient of -4.026 significant at the 1% level controlling for the mean of the I/H spread, firm characteristics, and backward-looking belief heterogienity in Column (1) of Table X. The  $\mu_{I/H,P}$ , the mean of the volatility premium, also has significant cross-sectional explanatory power at the 5% level, consistent with previous findings (Bali and Hovakimian, 2009).

Notably,  $\sigma_{I/H,P}$  has explanatory power for the cross-section of stock returns in excess of  $\mu_{I/H,P}$ , IDIOVOL, and other controls suggesting that it contains unique price information.

Column (2) of Table X presents analogous results for  $\sigma_{\Delta IV,P}$ . The standard deviation of IV innovation has cross-sectional explanatory power with a coefficient of -9.343 significant at the 1% level, controlling for  $\mu_{\Delta IV,P}$ , IDIOVOL, and other firm characteristics. Consistent with An, Ang, Bali and Cakici (2014),  $\mu_{\Delta IV,P}$  has a cross-sectional negative coefficient of -2.626. Its insignificance, together with Table VIII, indicates that the predictive power of  $\sigma_{\Delta IV,P}$  dominates that of  $\mu_{\Delta IV,P}$ . These findings confirm that  $\sigma_{\Delta IV,P}$  has explanatory power for the cross-section of stock returns in excess of that available from the average level of the IV innovation and other control variables, and further underscores the importance of the variability in the levels of these variables.

Column (3) of Table X presents the cross-sectional findings for  $\sigma_{IV,P}$  in the full sample. In Column (3)  $\sigma_{IV,P}$  has a coefficient of -4.585 significant at the 1% level, while the coefficient on  $\mu_{IV,P}$  is -1.754 at the 5% significant level, consistent with implied volatility levels indicative of fear (Whaley, 2000).

The coefficients on idiosyncratic volatility IDIOVOL as a historical proxy for heterogeneous beliefs (Friesen, Zhang, Zorn, 2012) are insignificant in all three columns, indicating that historical belief heterogeneity does not explain the cross-section of returns after controlling for our innovative measures.<sup>6</sup> Furthermore, this suggests that if  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$  are proxies of belief heterogeneity, the variability in forward-looking expectations makes them a superior proxy.

Taken together, the evidence in Table X presents strong evidence that the standard deviations of IV measures contain explanatory power for the cross-section of stock returns in excess of the mean levels of the IV measures themselves. These findings are also robust to controls for firm size, book-to-market ratio, historical volatility, option liquidity and stock

 $<sup>^{6}</sup>$ We do not include another heterogeneous belief proxy used by Friesen, Zhang, and Zorn (2012), the dispersion of analyst forecasts, due to the limited number of available observations due to requiring analyst data.

liquidity. We next investigate whether they can be related to heterogeneous beliefs and what drives these IVF variability measures.

# 4 Heterogeneous Beliefs and $\sigma$

I/H,  $\Delta IV$ , and IV reflect market expectations about the underlying. Their standard deviations over the prior month,  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ , and  $\sigma_{IV,P}$ , are by definition measures of variability in these expectations, and have predictive power for returns consistent with prior theory about the effects of heterogeneous beliefs (Miller, 1977). Thus, we next test whether the  $\sigma$  measures are indeed proxies of heterogeneous beliefs.

#### 4.1 **Prior Proxies of Heterogeneous Beliefs**

In this section we focus on the relation between standard deviations of IV measures and other known proxies of heterogeneous beliefs in the cross-section of stocks. Following Friesen, Zhang and Zorn (2012), we focus on DISPERSION, the dispersion in financial analysts' earnings forecasts, and IDIOVOL, the idiosyncratic volatility.

We match our data with monthly dispersion of financial analysts' forecasts from Institutional Brokers' Estimate System (IBES). This matching requirement reduces our sample to 64,296 firm-month observations during the period from January 1996 to August 2015. The median firm size of the sample is \$2.40 million compared to \$2.18 million of our full sample, suggesting that this subsample with analyst data is similar to the full sample previously considered. DISPERSION is measured as the standard deviation of forecasts for quarterly EPS scaled by mean EPS. IDIOVOL is calculated by regressing monthly stock returns over the prior 60 months on the FFCP5 model with market, size, value, momentum, and liquidity risk factors. In addition to estimating idiosyncratic volatility, we also retain the coefficient on market risk premium from this regression as the BETA, a control variable following Dennis and Mayhew (2002). Taylor et al. (2009) show that option and stock liquidity affects the risk-neutral return distribution for individual firms, requiring liquidity controls. We include the monthly average volume and open interest of ATM put options. Following Friesen, Zhang, and Zorn (2012), we also control for the underlying stock's bid-ask spread as a proxy for liquidity, as well as its leverage.

First, we conduct a univariate cross-sectional analysis of the relationship between  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$  and  $\sigma_{IV,P}$  with the belief heterogeneity proxies at the monthly frequency. Each month in our sample we run a cross-sectional regression across all firms *i* of the form

$$\sigma_i = \alpha_0 + \alpha_1 PROXY_i + \epsilon_i \tag{5}$$

where  $\sigma$  is the standard deviation of each of I/H,  $\Delta IV$ , and IV, and PROXY is one of the two variables representing heterogeneous beliefs: DISPERSION, the dispersion in financial analysts' EPS forecast, and IDIOVOL, the stock's idiosyncratic volatility. Table XI Panel A presents the results.

Columns (1) and (2) in Panel A of Table XI show the univariate cross-sectional relationship between  $\sigma_{I/H,P}$  and our two belief heterogeneity proxies. In Column (1) the cross-sectional coefficient on analyst dispersion is significant at 1% level with a value of 0.014, and in Column (2) for idiosyncratic stock volatility is also significant at the 1% significance level with a value of 0.204. Columns (3) and (4) present analogous findings for  $\sigma_{\Delta IV,P}$ . In Column (3) analyst dispersion has a coefficient of 0.008, and in Column (4) idiosyncratic volatility a coefficient of 0.160, both significant at the 1% level. Finally, Columns (5) and (6) of Table XI Panel A document the relationship between  $\sigma_{IV,P}$  and the belief heterogeneity proxies. Again both analyst dispersion and idiosyncratic volatility are significant at the 1% level with coefficients of 0.010, and 0.197 respectively.

The results in Table XI Panel A show that the three  $\sigma$  measures have a strong crosssectional relationship with analyst forecast dispersion and idiosyncratic stock volatility, which are documented proxies for belief heterogeneity (Friesen, Zhang, and Zorn, 2012). Next, we test whether this relationship is robust to the inclusion of controls for firm characteristics. We add them to our cross-sectional model as follows:

$$\sigma_{i} = \alpha_{0} + \alpha_{1} PROXY_{i} + \alpha_{2} LIQUIDITY_{i} + \alpha_{3} LEVERAGE_{i} + \alpha_{4} BETA_{i} + \alpha_{5} VOL_{p,i} + \alpha_{6} OI_{p,i} + \epsilon_{i}$$

$$(6)$$

Here  $\sigma$  is the standard deviation of corresponding IV measure, VOL is the monthly average of daily ATM put volume for the month, OI is the monthly average of daily option open interest, LEVERAGE is the debt to total asset ratio of the firm, and BETA is the firm's market beta from the FFCP5 model. PROXY is one of the two variables representing heterogeneous beliefs: the dispersion in financial analysts' EPS forecast and underlying idiosyncratic volatility. Table XI Panel B presents the relationship between  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$ and  $\sigma_{IV,P}$  and two proxies of heterogeneous beliefs in the presence of firm characteristics.

Columns (1) and (2) of Table XI Panel B show that the findings for the cross-sectional relationship between  $\sigma_{I/H,P}$  and our two belief heterogeneity proxies are not significantly affected by the inclusion of other firm characteristics. In Column (1) the cross-sectional coefficient for analyst dispersion has a value of 0.011, and in Column (2) idiosyncratic stock volatility has a value of 0.176, both significant at the 1% level. Columns (3) and (4) show the results for  $\sigma_{\Delta IV,P}$  controlling for firm characteristics. In Column (3) analyst dispersion has a coefficient of 0.006, and in Column (4) idiosyncratic volatility a coefficient of 0.152, both significant at the 1% level. Finally, Columns (5) and (6) of Table XI Panel B document the relationship between  $\sigma_{IV,P}$  and the belief heterogeneity proxies, with both analyst dispersion and idiosyncratic volatility significant at the 1% level with coefficients of 0.008, and 0.186 respectively. In all cases the coefficients in Panel B, controlling for firm characteristics, are only slightly smaller than their univariate values in Panel A.

The results in Table XI show that the three IVF variability measures have a strong cross-sectional relationship with analyst forecast dispersion and idiosyncratic stock volatility, which are documented proxies for belief heterogeneity (Friesen, Zhang, and Zorn, 2012). These findings suggest that  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$  and  $\sigma_{IV,P}$  are themselves proxies for investor belief heterogeneity in the cross-section of stocks. This, coupled with the cross-sectional pricing evidence from Table X, shows that the forward-looking  $\sigma$  variables are superior proxies of heterogeneous beliefs compared with IDIOVOL and DISPERSION.

#### 4.2 Determinants of $\sigma$

We have thus far established the predictive power of  $\sigma$ s for returns in excess of firm characteristics, and its superior information content relative to prior proxies of heterogeneity of investor beliefs. We next consider the determinants of  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$  and  $\sigma_{IV,P}$  using panel regression analysis to test their uniqueness. We first estimate the contemporaneous relationship between the monthly standard deviation  $\sigma$  and the mean  $\mu$  of each of the three option-based measures of investor beliefs for firm i in month t:

$$\sigma_{i,t} = a_0 + a_1 \mu_{i,t} + F E_t + \epsilon_{i,t} \tag{7}$$

We run the regression from January 1996 to August 2015 with month fixed effects  $FE_t$ . Table XII reports parameter estimates for the model in Eq. (7) in Column (1) for  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$  and  $\sigma_{IV,P}$  of Panels A, B, and C respectively. Consistent with the correlations in Table II, the coefficients on  $\mu_{I/H,P}$  and  $\mu_{IV,P}$  in Panel A and C are 0.0916 and 0.1268, and significant at the 1% level. In other words, firms with higher average IV spread between ATM puts and Realized Volatility and average IV have higher standard deviation of I/H spread and IV. For IV innovation in Panel B, the coefficient of  $\mu_{\Delta IV,P}$  is negative and significant at 1% level, indicating the higher level of IV innovation associated with lower variability of IV innovation.

We also test the time series properties of  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$  and  $\sigma_{IV,P}$  by estimating an

autoregressive panel model with time fixed effects:

$$\sigma_{i,t} = a_0 + a_2 \sigma_{i,t-1} + F E_t + \epsilon_{i,t} \tag{8}$$

Parameter estimates for regression equation 8 are reported in Column (2) of Table XII for  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$  and  $\sigma_{IV,P}$  in Panels A, B, and C respectively. The coefficients on  $\sigma_{i,t-1}$ are positive and significant at the 1% level and range from 0.34 for  $\sigma_{I/H,P}$  to 0.40 for  $\sigma_{\Delta IV,P}$ , indicating that the  $\sigma$  measures are only moderately persistent over time.

Next, we add firm and macroeconomic characteristics while controlling for the effects of  $\mu$  and the autocorrelation of  $\sigma$ :

$$\sigma_{i,t} = a_0 + a_1 \mu_{i,t} + a_2 \sigma_{i,t-1} + a_3 VOL_{i,t} + a_4 OI_{i,t} + a_5 MV_{i,t} + a_6 BM_{i,t} + a_7 SP500_t + a_9 CON_t + a_{10} EXP_t + \epsilon i, t$$
(9)

Here  $VOL_{i,t}$  and  $OI_{i,t}$  are the monthly averages of daily volume and open interest for ATM puts respectively,  $MV_{i,t}$  is the natural logarithm of the firm's market value, BM is the book to market ratio,  $SP500_{i,t}$  is value-weighted return (including dividend) on the S&P 500 index for the month , CON is an indicator dummy for economic contractions defined from 3/2001 to 11/2001 and from 12/2007 to 6/2009, and EXP is an analogous indicator for economic expansions from 1/1996 to 12/1999 and 1/2005 to 7/2007.

We report parameter estimates for Eq. (9) in Column (3) of Table XII. The sign and significance of coefficients on  $\mu_{i,t}$  and  $\sigma_{i,t-1}$  are consistent with prior results from Columns (1) and (2). The coefficients on MV, firm size, range from an insignificant -0.0203 for  $\sigma_{IV,P}$ to -0.8591 for  $\sigma_{I/H,P}$  significant at the 1% level in all cases indicating that smaller firms have higher  $\sigma$ . Furthermore, the coefficients on OI, put open interest, range from -.026 for  $\sigma_{\Delta IV,P}$  to -.0498 for  $\sigma_{IV,P}$ , all significant at the 1% level. As expected, smaller firms with more liquid option markets have more variability in the option-based predictive measures consistent with their higher information asymmetry (Karpoff, Lee, and Masulis, 2013) and therefore a larger heterogeneity of beliefs.

Meanwhile, the coefficients VOL, put option volume, range from .220 for  $\sigma_{\Delta IV,P}$  to .646 for  $\sigma_{I/H,P}$  significant at the 1% level. Open interest controls for the overall liquidity of the options market, and this finding on the positive relationship between volume and belief heterogeneity is consistent with findings by Buraschi and Jiltsov (2006). Overall, the determinants of  $\sigma$  are therefore also consistent with a heterogeneous beliefs interpretation.

## 5 Belief Heterogeneity and Short Sale constraints

The analysis in the preceding section supports the view that  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$  and  $\sigma_{IV,P}$  are positively related to heterogeneous beliefs. Furthermore, the negative relationship between  $\sigma$ s and future stock returns is consistent with the theorized negative relationship between belief heterogeneity and expected returns in Miller (1997). Notably, however, the findings of Miller (1997) are derived under the assumption of short sale constraints. In this section we test whether short sale constraints affect  $\sigma$ s and their predictive power for stock returns. We utilize the natural experiment conducted by the Securities and Exchange Commission (SEC), Regulation SHO (Reg SHO), following the approach of Fang, Huang, and Karpoff (2015). Reg SHO temporarily suspended short sale price tests for a set of designated pilot securities, a third of the Russell 3000 index, during 2005-2007. On May 2, 2005, the randomly selected pilot stocks began to trade without short sale price tests, with the remaining Russell 3000 securities remaining unaffected as controls. We create a subsample of pilot and control stocks from the intersection of the Russell 3000 constituents and our dataset. This sub-sample with data from 1996 to 2015 includes 158,390 firm-month observations, 2,042 firms and 688 pilot stocks.

First, we test whether the reduction in short-sale constraints during the pilot phase of Reg SHO in 2005-2007 had an effect on  $\sigma_{I/H,P}$ ,  $\sigma_{\Delta IV,P}$  and  $\sigma_{IV,P}$ . Our treatment group indicator,

SHO, is equal to one if the firm is included in the pilot program. The event indicator, EFF, is equal to one from 2005 to 2007. We follow the standard difference-in-differences model by including an interaction term between the treatment and event indicators as the experimental variable in studying the effects of short-sale constraints on  $\sigma$ :

$$\sigma_{i,t} = a_0 + a_1 \mu_{i,t} + a_2 \sigma_{i,t-1} + a_3 SHO \times EFF + a_4 SHO + a_5 EFF + a_6 M V_{i,t} + a_7 B M_{i,t} + a_8 VOL_{i,t} + a_9 OI_{i,t} + a_{10} SP500_t + \epsilon i, t$$
(10)

We include our standard battery of controls:  $\mu_{i,t}$  is the monthly average level of the predictive variable,  $\sigma_{i,t-1}$  is the lagged standard deviation of the variable,  $MV_{i,t}$  is the logarithm of market value, BM is the book to market ratio,  $VOL_{i,t}$  and  $OI_{i,t}$  are the monthly average of daily volume and open interest for ATM puts respectively, and  $SP500_{i,t}$  is value-weighted return (including dividend) on the S&P 500 index for the month.

Panel A Table XIII summarizes the difference-in-difference results. For  $\sigma_{I/H,P}$  and  $\sigma_{IV,P}$ , the difference-in-difference interaction terms are positive and significant at 5% level and 10% level respectively. The suspension of short sale price tests during Reg SHO increases  $\sigma_{I/H,P}$ and  $\sigma_{IV,P}$ . However, the difference-in-difference estimator of  $\sigma_{\Delta IV,P}$  are insignificant from zero, indicating that  $\sigma_{\Delta IV,P}$  are not affected by the short-sale constraints. The increase in some  $\sigma$  measures during the Reg SHO period is not surprising, since the reduction of shortsale constraints enables pessimistic market participants to more easily reflect their beliefs increasing overall belief heterogeneity.

Next, we test whether the reduction in short-sale constraints has an impact on the predictive power of  $\sigma$ s for stock returns. If short-sale constraints assumption in Miller (1977) is critical to the observed result, we should expect return predictability from  $\sigma$ s to worsen for the pilot stocks during Reg SHO as short-sale constraints are reduced. We regress excess returns on  $\sigma$ s and their interaction with *SHO* and *EFF* controlling for other predictive variables including firm size, *MV*, book-to-market, *BM*, realized volatility over the prior year, *RV*, and liquidity measured by ATM put option volume, *VOL*, and open interest, *OI*,

and monthly stock volume VOLUME. We also control for the mean level of the option-based predictive variables  $\mu$ s and idiosyncratic volatility IDIOVOL:

$$exret_{i,t+1} = a_0 + a_1\sigma_{i,t} + a_2\mu_{i,t} + a_3\sigma_{i,t} \times SHO \times EFF + a_4\sigma_{i,t} \times EFF + a_5\sigma_{i,t} \times SHO$$
$$+ a_6SHO \times EFF + a_7EFF + a_8SHO + a_9MV_{i,t} + a_{10}BM_{i,t}$$
$$+ a_{11}RV_{i,t} + a_{12}VOL_{i,t} + a_{13}OI_{i,t} + a_{14}VOLUME_{i,t} + a_{15}IDIOVOL_{i,t} + \epsilon i, t$$
(11)

Panel B Table XIII presents the panel regression results. We find that the coefficients on the interaction of SHO, EFF, and  $\sigma$ s are insignificant for all three predictive variables, meaning that the assumption of short-sale constraints does not seriously affect the return predictability from  $\sigma$ s as measures of belief heterogeneity. Consistent with prior results, the standard deviation of the I/H spread,  $\sigma_{I/H,P}$ , the standard deviation of IV innovation,  $\sigma_{\Delta IV,P}$ and the standard deviation of implied volatility of ATM puts,  $\sigma_{IV,P}$  have strong negative coefficients -14.164, -8.028 and -19.297 respectively and all of them are significant at the 1% level.

#### 6 Conclusion

The volatility premiums, implied volatility innovations, and implied volatility levels reflect investor beliefs about the underlying asset (Bali and Hovakimian, 2009; An, Ang, Bali, and Cakici, 2014). The standard deviations of these belief measures should reflect variability in these beliefs. Two competing views in the literature, the Miller (1977) overvaluation theory and the Williams (1977) risk theory, predict diametrically opposite price effects for variability in investor beliefs. If the standard deviations of investor beliefs reflect their variability, they should therefore have predictive power for stock returns. Our results are consistent with this implication.

We find that the standard deviations of these three option-based measures of investor

beliefs has significant predictive power for negative future performance of the underlying asset. This inverse relationship between variability of belief measures and returns is not due to the correlation between the standard deviations and firm size, book-to-market ratios, or the levels of the option-based investor expectation measures themselves. This implies that our results are not due to the predictability effects documented for the levels of these measures in prior literature by Goyal and Saretto (2009), Bali and Hovakimian, (2009), and An, Ang, Bali and Cakici (2014).

Indeed, we find that the standard deviations of these option-based predictors of stock returns are more significantly and robustly priced in the cross-section of stock returns than their previously-studied levels. These standard deviations exhibit a consistently negative price impact in both portfolio sorts and in the cross-section of returns relative to the Fama and French (1993), Carhart (1997), and Pastor and Stambaugh (2005) risk factor models, as well as a battery of firm characteristic controls.

Furthermore, we find that the standard deviations of IV spreads are highly significantly related to analyst dispersion and idiosyncratic stock volatility, documented proxies of heterogeneous beliefs (Friesen, Zhang, and Zorn, 2012). This is consistent with the overvaluation theory of Miller (1977), supporting belief heterogeneity as the cause of the observed negative return predictability and cross-sectional price effect of standard deviations of option-based measures of investor beliefs. Consistent with this interpretation, we find that the magnitudes of these belief variability measures are negatively related to firm size and liquidity, consistent with information asymmetry and belief heterogeneity. They are positively related with option volume, consistent with the findings on the relationship between volume and belief heterogeneity by Buraschi and Jiltsov (2006). Finally, we use an exogenous shock to short-sale constraints in the SEC implementation of the Reg SHO pilot program in 2005 to show that the return predictability from our standard deviation measures is not affected by a reduction in short-sale constraints.

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# Appendix A

We detail the construction of our variables below. Summary statistics of these variables are reported in Table I.

 $\mu_{I/H,P}$ 

Monthly average of daily spread between IV of at-the-money puts and realized volatility over the past year. Following Bollen and Whaley (2004), a put option is defined as ATM when the delta of the option is between -0.625 and -0.375.

 $\sigma_{I/H,P}$ 

Monthly standard deviation of daily spread between IV of at-the-money puts and realized volatility over the past year. Following Bollen and Whaley (2004), a put option is defined as ATM when the delta of the option is between -0.625 and -0.376.

 $\mu_{\Delta IV,P}$ 

Monthly average of daily implied volatility innovation, which is defined as the first difference of daily average implied volatility of at-the-money puts. Following Bollen and Whaley (2004), a put option is defined as ATM when the delta of the option is between -0.625 and -0.375.

 $\sigma_{\Delta IV,P}$ 

Monthly standard deviation of daily implied volatility innovation, which is defined as the first difference of daily average implied volatility of at-the-money puts. Following Bollen and Whaley (2004), a put option is defined as ATM when the delta of the option is between -0.625 and -0.375.

 $\mu_{IV,P}$ 

Monthly average of daily average implied volatility of at-the-money puts. Following Bollen and Whaley (2004), a put option is defined as ATM when the delta of the option is between -0.625 and -0.375.

 $\sigma_{I/H}$ 

Monthly standard deviation of daily average implied volatility of at-the-money puts. Following Bollen and Whaley (2004), a put option is defined as ATM when the delta of the option is between -0.625 and -0.375.

MV

Log market capitalization

BM

Log book to market ratio

VOLUME

Monthly stock volume

SP500

Monthly value-weighted return of SP500 index (including dividend)

 $\mathrm{RV}$ 

Realized volatility over past year

LEVERAGE

The debt ratio from the firm

#### BASPREAD

Bid-Ask Spread on the firm stock scaled by ask price.

#### BETA

The firm's beta is the coefficient on market risk premium from the regression of excess monthly stock returns over the last 60 months on the Fama-French (1993) three factors, Carhart (1997) momentum factor plus Pastor and Stambaugh (2003) liquidity factor five factors.

#### IDIOVOL

The idiosyncratic volatility is standard deviation of residuals from the regression of excess monthly stock returns over the last 60 months on the Fama-French (1993) three factors, Carhart (1997) momentum factor plus Pastor and Stambaugh (2003) liquidity factor five factors. We also obtain the beta from the regression.

#### DISPERSION

The dispersion in financial analysts' earnings forecast. Dispersion is measured as the standard deviation of forecasts for quarterly earnings, scaled by the mean of forecasts. The data on financial analysts' earnings forecasts are taken from Institutional Brokers' Estimate System detail history data set. Only the last forecast is kept. Firms with a zero mean forecast or without a standard deviation are excluded.

#### $VOL_{P,ATM}$

The monthly average of daily volume of ATM put options.

 $OI_{P,ATM}$ 

The monthly average of daily open interests of ATM put options.

 $CON_t$ 

A dummy variable which is equal to 1 if the date of observation is from Mar 2001 to Nov 2001 or from Dec 2007 to Jun 2009, and equal to 0 otherwise.

 $EXP_t$ 

A dummy variable which is equal to 1 if the date of observation is from Jan 1996 to Dec 1999, or from Jan 2005 to Jul 2007, and equal to 0 otherwise.

Table I: Descriptive Statistics. This table provides the descriptive statistics of means and standard deviations of three implied volatility measures, as well as of the firm-specific variables that are used in subsequent analysis. The sample consists of 238,847 firm-month combinations, constituting monthly observations from Jan 1996 through Aug 2015 from OptionMetrics, Compustat and CRSP. Individual variable definitions are outlined in the Appendix.

37 . 11	NT	Dr	DF0	DOF	1.0	(TTD)
Variables	N	P5	P50	P95	Mean	STD
$\mu_{I/H,P}$	$238,\!847$	-0.222068	0.005089	0.207049	0.001750	0.149428
$\sigma_{I/H,P}$	$238,\!847$	0.007021	0.030673	0.117753	0.043312	0.046880
$\mu_{\Delta IV,P}$	$238,\!847$	-0.018485	-0.000334	0.015283	-0.000755	0.016262
$\sigma_{\Delta IV,P}$	$238,\!847$	0.016141	0.036947	0.101317	0.045586	0.035481
$\mu_{IV,P}$	$238,\!847$	0.209176	0.429214	0.936460	0.482837	0.235717
$\sigma_{IV,P}$	$238,\!847$	0.007563	0.032532	0.122890	0.045372	0.046891
IDIOVOL	$238,\!847$	0.048628	0.106461	0.241231	0.121310	0.069172
MV	$238,\!847$	$268,\!143$	$2,\!176,\!930$	$35,\!011,\!496$	$8,\!862,\!238$	26,073,899
BM	$238,\!847$	0.062043	0.330341	1.529217	0.944471	10.161572
RV	$238,\!847$	0.200689	0.423239	0.954775	0.480270	0.243279
VOLUME	$238,\!847$	$28,\!398$	$169,\!809$	$1,\!409,\!010$	$397,\!821$	841,711
$VOL_{P,ATM}$	$238,\!847$	6.5	42.5	448.6	120.0	295.2
$OI_{P,ATM}$	$238,\!847$	19.3	274.6	$3,\!476.2$	898.3	2,465.4
LEVERAGE	$64,\!296$	0	0.151132	0.516091	0.181451	0.176159
BASPREAD	$64,\!296$	0.000109	0.000939	0.016484	0.003710	0.011713
BETA	$64,\!296$	0.116261	1.129346	2.526855	1.183543	1.080260
DISPERSION	$64,\!296$	0.011034	0.065482	0.872270	0.313970	2.552549
MV (sub sample)	$64,\!296$	$308,\!434$	$2,\!400,\!870$	$37,\!489,\!219$	$9,\!498,\!984$	$27,\!424,\!068$

(10) $(11)$ $(12)$ $(13)$	0.973 0.058 0.039 0.013	0.12.0 CTU.U 26U.U 26U.U CTU.U	-0.273 $-0.036$ $0.126$ $0.0590.495$ $-0.001$ $0.126$ $0.059$	-0.27 $-0.092$ $0.012$ $0.0130.495$ $-0.001$ $0.126$ $0.059-0.025$ $0.010$ $-0.008$ $0.010$	-0.27 $-0.095$ $0.045$ $0.0100.495$ $-0.001$ $0.126$ $0.059-0.025$ $0.010$ $-0.008$ $0.0100.469$ $-0.208$ $-0.094$ $-0.138$	-0.273 $-0.036$ $0.032$ $0.0130.495$ $-0.001$ $0.126$ $0.059-0.025$ $0.010$ $-0.008$ $0.0100.469$ $-0.208$ $-0.094$ $-0.1380.896$ $-0.098$ $-0.002$ $-0.032$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$						
(6)	0.040	-0.040	-0.002	-0.001	-0.027	-0.051	-0.095	-0.185	1.000	-0.044	-0.034	-0.024	)
(8)	-0.027	-0.351	0.038	-0.494	-0.650	-0.335	-0.651	1.000	-0.037	-0.260	0.546	0.442	111
(2)	-0.123	0.444	-0.033	0.498	0.801	0.449	1.000	-0.253	-0.058	0.725	-0.080	-0.066	0.070
(9)	0.130	0.954	-0.052	0.416	0.588	1.000	0.329	-0.125	-0.019	0.398	0.010	0.052	0.014
(5)	0.105	0.578	-0.030	0.517	1.000	0.575	0.678	-0.273	-0.039	0.827	-0.041	-0.030	0.000
(4)	0.054	0.421	-0.066	1.000	0.420	0.411	0.335	-0.172	-0.011	0.336	-0.103	-0.061	0.076
(3)	-0.005	-0.053	1.000	-0.072	-0.017	-0.042	-0.020	0.012	-0.001	-0.013	0.006	0.001	0000
(2)	0.125	1.000	-0.043	0.408	0.556	0.957	0.324	-0.127	-0.018	0.391	-0.000	0.039	0000
(1)	1.000	0.256	-0.001	0.124	0.252	0.277	-0.114	-0.011	0.015	-0.298	-0.027	0.027	0.010
	$(1)\mu_{I/H,P}$	$(2)\sigma_{I/H,P}$	$(3)\mu_{\Delta IV,P}$	$(4)\sigma_{\Delta IV,P}$	$(5)\mu_{IV,P}$	$(6)\sigma_{IV,P}$	(1)IDIONOF	(8)MV	(9)BM	$(10) \mathrm{RV}$	(11)VOLUME	$(12)VOL_{P,ATM}$	(19) OT

	$\sigma_{I/H,P}$	$\sigma_{\Delta IV,P}$	$\sigma_{IV,P}$	$\mu_{I/H,P}$	$\mu_{\Delta IV,P}$	$\mu_{IV,P}$
1% SIG 5% SIG 10% SIG TOTAL	$76.84\% \\ 5.26\% \\ 3.12\% \\ 85.23\%$	$76.76\% \\ 6.27\% \\ 2.25\% \\ 85.28\%$	$76.47\% \\ 6.16\% \\ 3.32\% \\ 85.95\%$	83.00% 3.73% 2.23% 88.96%	$79.24\% \\ 5.46\% \\ 2.49\% \\ 87.19\%$	83.55% 2.49% 1.88% 89.39%

Table III: Augmented Dickey Fuller Tests.

Table IV: Descriptive Statistics: Portfolios Formed on Standard Deviations of Implied Volatility Measures ( $\sigma$ s). Panel A and Panel B present results for equal-weighted and value-weighted portfolios sorted by magnitude of standard deviations of the three option-based investor belief measures respectively. The table reports excess returns along with abnormal performance relative to standard benchmarks. We benchmark performance using the Capital Asset Pricing Model (CAPM Alpha), Fama and French (1993) 3-factor model (FF3 Alpha), Fama and French (1993) 3 factors plus Carhart (1997) momentum factor 4-factors model (FFC4 Alpha) and Fama and French (1993) 3 factors, Carhart (1997) momentum factor plus Pastor and Stambaugh (2003) liquidity factor 5-factor model (FFC5 Alpha) over the month following portfolio formation.

	1	9	σQι	uintiles 4	5	$t(5_{-}1)$				
$\sigma_{I/H,P}$		2	0	T	0	0(0-1)				
	1.07%	1 10%	1.06%	0.80%	0.53%	-0.54%				
Excess Return	(3.30)	(2.84)	(2.53)	(1.61)	(0.95)	(-1.54)				
	0.34%	0.26%	0.13%	-0.24%	-0.61%	-0.96%***				
CAPM Alpha	(2.13)	(1.68)	(0.64)	(-1.01)	(-2.17)	(-3.03)				
	0.25%	0.10%	0.09%	-0.25%	-0.61%	-0.86%***				
FF3 Alpha	(2.25)	(1.56)	(0.56)	(-1.58)	(-3.78)	(-5.11)				
	0.28%	0.25%	0.19%	-0.10%	-0.38%	-0.66%***				
FFC4 Alpha	(2.46)	(2.05)	(1.23)	(-0.68)	(-2.57)	(-3.82)				
	0.30%	0.25%	0.20%	-0.10%	-0.38%	-0.68%***				
FFCP5 Alpha	(2.60)	(2.04)	(1.19)	(-0.58)	(-2.47)	(-3.90)				
					· · · ·					
$\sigma_{\Delta IV,P}$										
	0.98%	1.18%	1.12%	0.82%	0.45%	$-0.53\%^{*}$				
Excess Return	(3.05)	(2.99)	(2.54)	(1.65)	(0.85)	(-1.76)				
	0.25%	0.32%	0.14%	-0.22%	-0.62%	-0.87%***				
CAPM Alpha	(1.85)	(1.79)	(0.72)	(-0.95)	(-2.27)	(-3.15)				
	0.16%	0.24%	0.11%	-0.21%	-0.63%	$-0.80\%^{***}$				
FF3 Alpha	(1.56)	(1.68)	(0.69)	(-1.43)	(-3.96)	(-5.15)				
	0.20%	0.33%	0.23%	-0.06%	-0.48%	$-0.68\%^{***}$				
FFC4 Alpha	(1.91)	(2.40)	(1.58)	(-0.40)	(-3.24)	(-4.14)				
	0.21%	0.36%	0.25%	-0.05%	-0.49%	$-0.70\%^{***}$				
FFCP5 Alpha	(1.93)	(2.45)	(1.59)	(-0.34)	(-3.17)	(-4.20)				
$\sigma_{IV,P}$										
	1.04%	1.12%	1.02%	0.81%	0.56%	-0.48%				
Excess Return	(3.14)	(3.02)	(2.42)	(1.62)	(1.00)	(-1.38)				
	0.31%	0.29%	0.09%	-0.24%	-0.58%	$-0.90\%^{***}$				
CAPM Alpha	(1.96)	(1.77)	(0.47)	(-1.04)	(-2.03)	(-2.76)				
	0.21%	0.22%	0.06%	-0.24%	-0.58%	$-0.79\%^{***}$				
FF3 Alpha	(1.89)	(1.82)	(0.34)	(-1.53)	(-3.59)	(-4.60)				
	0.25%	0.27%	0.16%	-0.09%	-0.35%	$-0.60\%^{***}$				
FFC4 Alpha	(2.15)	(2.16)	(1.08)	(-0.61)	(-2.39)	(-3.37)				
	0.27%	0.28%	0.18%	-0.08%	-0.36%	$-0.62\%^{***}$				
FFCP5 Alpha	(2.25)	(2.08)	(1.12)	(-0.50)	(-2.34)	(-3.47)				

		$\sigma$ Quintiles							
	1	2	3	4	5	t(5-1)			
$\sigma_{I/H,P}$									
Excess Return	0.84% (3.37)	0.92% (2.79)	0.79% (2.09)	0.80% (1.82)	$0.33\% \ (0.55)$	-0.51% (-1.13)			
CAPM Alpha	0.26%	0.20%	-0.04%	-0.15%	-0.74%	$-1.00\%^{**}$			
	(2.52)	(1.92)	(-0.38)	(-0.81)	(-2.57)	(-2.76)			
FF3 Alpha	0.25%	0.22%	0.01%	-0.07%	-0.62%	$-0.87\%^{*3}$			
	(2.91)	(2.21)	(0.11)	(-0.45)	(-3.42)	(-3.81)			
FFC4 Alpha	0.20% (2.14)	0.18% (1.89)	$0.02\% \ (0.14)$	-0.02% (-0.15)	-0.52% (-2.76)	$-0.72\%^{*}$ (-2.94)			
FFCP5 Alpha	0.23%	0.19%	-0.00%	-0.03%	-0.53%	$-0.76\%^{*3}$			
	(2.51)	(2.02)	(-0.01)	(-0.16)	(-2.72)	(-3.04)			
$\sigma_{\Delta IV,P}$									
Excess Return	0.77% (2.82)	0.97% (2.58)	0.86% (1.87)	0.51% (1.03)	0.41% (0.84)	-0.37% (-1.21)			
CAPM Alpha	0.14%	0.17%	-0.08%	-0.45%	-0.56%	$-0.70\%^{*1}$			
	(1.56)	(1.51)	(-0.46)	(-2.42)	(-2.85)	(-2.92)			
FF3 Alpha	0.16%	0.20%	-0.01%	-0.37%	-0.55%	$-0.71\%^{*}$			
	(2.04)	(1.76)	(-0.06)	(-2.41)	(-3.09)	(-3.54)			
FFC4 Alpha	0.14%	0.19%	0.02%	-0.32%	-0.45%	$-0.60\%^{*}$			
	(1.89)	(1.59)	(0.14)	(-2.16)	(-2.94)	(-3.35)			
FFCP5 Alpha	0.15%	0.22%	0.02%	-0.32%	-0.46%	$-0.61\%^{*}$			
	(1.89)	(1.77)	(0.15)	(-2.01)	(-2.85)	(-3.29)			
$\sigma_{IV,P}$									
Excess Return	0.81%	0.89%	0.89%	0.86%	0.47%	-0.34%			
	(3.18)	(2.88)	(2.33)	(1.96)	(0.77)	(-0.73)			
CAPM Alpha	0.23%	0.19%	0.06%	-0.10%	-0.64%	$-0.87\%^{*}$			
	(1.99)	(1.66)	(0.49)	(-0.57)	(-2.19)	(-2.31)			
FF3 Alpha	0.21%	0.20%	0.11%	-0.01%	-0.51%	$-0.72\%^{*}$			
	(2.25)	(1.93)	(0.94)	(-0.05)	(-2.87)	(-3.19)			
FFC4 Alpha	0.16%	0.15%	0.11%	0.06%	-0.40%	$-0.56\%^{*}$			
	(1.61)	(1.55)	(1.00)	(0.33)	(-2.24)	(-2.41)			
FFCP5 Alpha	0.19%	0.17%	0.12%	0.04%	-0.43%	$-0.62\%^{*}$			
	(1.94)	(1.67)	(0.95)	(0.21)	(-2.31)	(-2.64)			

	'n																
$^{o,ATM}$	Media		742 901	842 842	824	784		1016	978	792	089 549		697	807	793	817	000
$OI_1$	Mean		893 1 048	1,046 1.046	096	885		1,279	1,158	969 700	598 598		853	1038	1,035	982	000
P,ATM	Median		99 117	11/ 120	114	104		132	134	116	94 74		96	114	115	117	100
NOL	Mean		116	$140 \\ 139$	129	114		160	157	132	001 79		112	137	139	133	077
JME	Median		372,367 445 859	440,002 $455,928$	423,514	346,770		496,223	511,090	446,704	301,094 $234,791$		354,061	437,857	448,276	430, 397	010 100
NOLU	Mean		374,148 144	454,455 $459,814$	432,251	366, 147		516,434	521,956	446,811	301,007 233,950		358,702	444,952	457,308	442,836	000 1000
N	Median		0.33	0.38 0.45	0.53	0.60		0.32	0.41	0.48	$0.52 \\ 0.58$		0.33	0.38	0.46	0.52	
щ	Mean		0.36	$0.42 \\ 0.48$	0.55	0.64		0.36	0.43	0.50	0.63		0.36	0.41	0.48	0.55	200
M	Median		0.84	0.72 0.67	0.61	0.56		0.81	0.79	0.70	0.52		0.87	0.71	0.66	0.59	( ) )
В	Mean		0.99	0.78	0.81	0.73		0.96	0.87	0.86	0.79		1.05	0.87	0.80	0.77	
	Median		15,498,116	13,001,779 9,353,347	5,788,798	2,876,248		23,200,880	11,176,109	6,216,468	3,000,191 $1,846,114$		15,166,859	13,578,275	9, 375, 565	5,898,693	
M	Mean		16,009,687	14,234,209 $9,709,307$	6,120,916	3,128,490		23, 343, 103	12,159,971	6,610,406	4,150,575 $1,955,176$		15,470,878	14, 135, 144	10,004,918	6,345,401	
	Median		0.004049	-0.007176	-0.003902	0.001591		-0.000111	-0.000215	-0.000541	-0.000889		0.328308	0.372355	0.437439	0.497355	0001000
Щ	Mean		0.000977	-0.008660	-0.010077	0.001617		-0.000287	-0.000304	-0.000575	-0.000109 -0.001128		0.362271	0.409500	0.474138	0.542816	
	Quintile	$\sigma_{I/H,P}$	-1 c	v 00	4	ល	$\sigma_{\Delta IV,P}$	H.	2	თ	4 ro	$\sigma_{IV,P}$	1	2	3	4	r

Table V: Descriptive Statistics: Portfolios Formed on Standard Deviations of Implied Volatility Measures ( $\sigma$ s). This table reports mean and median firm characteristics of quintile portfolios formed on  $\sigma$ s.

Table VI: Double-Sorted Portfolios by Size and  $\sigma$ s. For each  $\sigma$  measure in the left panel we net out the influence of size. We first sort firms by market capitalization into 10 portfolios and then within each size decile sort firms into two portfolios by corresponding  $\sigma$ . We then average the one-period returns across all size-sorted portfolios to create returns of two portfolios with similar levels of size but different  $\sigma$ . In right panel we reverse this procedure, and first sort firms by  $\sigma$  into 10 portfolios, and then within each  $\sigma$  portfolio sort firms by size into two portfolios. We then average the one-period returns across all  $\sigma$ -sorted portfolios to create returns of  $\sigma$  but different size. We report the differences across the two conditionally sorted portfolios in both left and right panels. The t-statistics are reported in parentheses and adjusted following Newey and West (1987) with a lag of 6 months.

	C	ontrolling $\sigma$ Ran	for Size k	С	ontrolling Size Ran	for $\sigma$ k
	Low	High	High-Low	Low	High	High-Low
$\sigma_{I/H,P}$						
Excess Return	1.05% (2.80)	0.77% (1.60)	-0.28% (-1.64)	0.96% (2.03)	0.86% (2.22)	-0.10% (-0.59)
CAPM Alpha	0.21%	-0.26%	-0.47%***	-0.05%	-0.00%	0.05%
	(1.22)	(-1.28)	(-3.00)	(-0.19)	(-0.01)	(0.27)
FF3 Alpha	0.13%	-0.27%	-0.40%***	-0.11%	-0.02%	0.09%
	(1.09)	(-2.01)	(-4.27)	(-0.75)	(-0.19)	(0.94)
FFC4 Alpha	0.21% (1.76)	-0.11% (-0.91)	-0.32%*** (-3.23)	$0.05\% \ (0.36)$	0.04% (0.42)	-0.01% (-0.12)
FFCP5 Alpha	0.23%	-0.11%	-0.34%***	0.07%	0.04%	-0.02%
	(1.81)	(-0.85)	(-3.31)	(0.43)	(0.43)	(-0.21)
$\sigma_{\Delta IV,P}$						
Excess Return	1.15%	0.68%	-0.47%***	1.07%	0.75%	-0.32%*
	(2.94)	(1.44)	(-2.71)	(2.28)	(1.90)	(-1.78)
CAPM Alpha	0.27%	-0.32%	$-0.59\%^{***}$	0.07%	-0.12%	-0.19%
	(1.45)	(-1.64)	(-3.61)	(0.28)	(-1.03)	(-1.01)
FF3 Alpha	0.18%	-0.31%	-0.49%***	-0.01%	-0.13%	-0.12%
	(1.28)	(-2.56)	(-4.31)	(-0.05)	(-1.26)	(-0.87)
FFC4 Alpha	0.30%	-0.21%	-0.52%***	0.17%	-0.07%	-0.24%*
	(2.35)	(-1.78)	(-4.73)	(1.06)	(-0.77)	(-1.86)
FFCP5 Alpha	0.33%	-0.22%	-0.54%***	0.19%	-0.07%	-0.26%*
	(2.42)	(-1.70)	(-4.91)	(1.13)	(-0.73)	(-1.96)

	Ce	ontrolling $\sigma$ Ran	for Size k	Controlling for $\sigma$ Size Rank				
	Low	High	High-Low	Low	High	High-Low		
$\sigma_{IV,P}$								
Excess Return	1.04%	0.79%	-0.25%	0.97%	0.86%	-0.11%		
	(2.77)	(1.62)	(-1.38)	(2.04)	(2.21)	(-0.62)		
CAPM Alpha	0.20%	-0.25%	-0.45%***	-0.04%	-0.01%	0.04%		
	(1.13)	(-1.20)	(-2.67)	(-0.17)	(-0.05)	(0.20)		
FF3 Alpha	0.12%	-0.25%	-0.37%***	-0.11%	-0.02%	0.09%		
	(0.95)	(-1.87)	(-3.71)	(-0.73)	(-0.21)	(0.89)		
FFC4 Alpha	0.20%	-0.10%	-0.30%***	0.06%	0.04%	-0.02%		
	(1.64)	(-0.80)	(-2.83)	(0.37)	(0.40)	(-0.15)		
FFCP5 Alpha	0.21%	-0.10%	-0.31%***	0.07%	0.04%	-0.03%		
	(1.67)	(-0.73)	(-2.88)	(0.46)	(0.39)	(-0.27)		

Table VII: Double-Sorted Portfolios by Book-to-Market Ratio and  $\sigma$ s. For each  $\sigma$  measure in the left panel we net out the influence of the book-to-market ratio. We first sort firms by book-to-market ratio into 10 portfolios and then within each book-to-market ratio decile sort firms into two portfolios by corresponding  $\sigma$ . We then average the one-period returns across all book-to-market ratio-sorted portfolios to create returns of two portfolios with similar levels of book-to-market ratio but different  $\sigma$ . In right panel we reverse this procedure, and first sort firms by  $\sigma$  into 10 portfolios, and then within each  $\sigma$  portfolio sort firms by bookto-market ratio into two portfolios. We then average the one-period returns across all  $\sigma$ -sorted portfolios to create returns of two portfolios with similar levels of  $\sigma$  but different book-to-market ratio. We report the differences across the two conditionally sorted portfolios in both left and right panels. The t-statistics are reported in parentheses and adjusted following Newey and West (1987) with a lag of 6 months.

	С	ontrolling $\sigma$ Ran	for BM k	C	ontrolling BM Ran	for $\sigma$ k
	Low	High	High-Low	Low	High	High-Low
$\sigma_{I/H,P}$						
Excess Return	1.08%	0.74%	-0.34%*	0.81%	1.01%	0.19%
	(2.96)	(1.51)	(-1.94)	(1.90)	(2.21)	(0.76)
CAPM Alpha	0.26%	-0.31%	-0.57%***	-0.10%	0.05%	0.15%
	(1.83)	(-1.36)	(-3.57)	(-0.51)	(0.20)	(0.57)
FF3 Alpha	0.20%	-0.33%	-0.52%***	-0.06%	-0.07%	-0.01%
	(1.85)	(-2.21)	(-5.45)	(-0.62)	(-0.40)	(-0.08)
FFC4 Alpha	0.24%	-0.15%	-0.39%***	-0.08%	0.17%	$0.25\%^{*}$
	(2.30)	(-1.05)	(-3.77)	(-0.84)	(1.10)	(1.89)
FFCP5 Alpha	0.25%	-0.14%	-0.39%***	-0.08%	0.19%	$0.27\%^{*}$
	(2.30)	(-0.94)	(-3.72)	(-0.80)	(1.13)	(1.93)
$\sigma_{\Delta IV,P}$						
Excess Return	1.09%	0.73%	-0.36%*	0.81%	1.01%	0.21%
	(2.96)	(1.49)	(-1.84)	(1.86)	(2.24)	(0.80)
CAPM Alpha	0.26%	-0.30%	-0.56%***	-0.11%	0.06%	0.18%
	(1.74)	(-1.30)	(-3.07)	(-0.58)	(0.27)	(0.67)
FF3 Alpha	0.18%	-0.32%	-0.50%***	-0.07%	-0.07%	0.00%
	(1.59)	(-2.15)	(-4.36)	(-0.69)	(-0.36)	(0.02)
FFC4 Alpha	0.26%	-0.16%	-0.42%***	-0.09%	0.18%	$0.27\%^{**}$
	(2.31)	(-1.13)	(-3.35)	(-0.91)	(1.16)	(2.04)
FFCP5 Alpha	0.27%	-0.16%	-0.43%***	-0.09%	0.20%	$0.29\%^{**}$
	(2.33)	(-1.06)	(-3.40)	(-0.87)	(1.19)	(2.07)

	Со	ontrolling $\sigma$ Ran	for BM k	Controlling for $\sigma$ BM Rank				
	Low	High	High-Low	Low	High	High-Low		
$\sigma_{IV,P}$								
Excess Return	1.08%	0.74%	-0.34%*	0.82%	1.00%	0.18%		
	(2.97)	(1.50)	(-1.81)	(1.92)	(2.19)	(0.74)		
CAPM Alpha	0.27%	-0.31%	-0.58%***	-0.09%	0.04%	0.14%		
	(1.82)	(-1.36)	(-3.33)	(-0.48)	(0.18)	(0.54)		
FF3 Alpha	0.19%	-0.32%	-0.52%***	-0.05%	-0.08%	-0.03%		
	(1.84)	(-2.17)	(-5.22)	(-0.54)	(-0.45)	(-0.16)		
FFC4 Alpha	0.24%	-0.14%	-0.38%***	-0.07%	0.16%	$0.24\%^{*}$		
	(2.30)	(-1.02)	(-3.64)	(-0.74)	(1.07)	(1.85)		
FFCP5 Alpha	0.25%	-0.14%	-0.38%***	-0.07%	0.18%	$0.26\%^{*}$		
	(2.28)	(-0.91)	(-3.59)	(-0.73)	(1.12)	(1.92)		

Table VIII: Double-Sorted Portfolios by  $\mu$ s and  $\sigma$ s. For each  $\sigma$  measure in the left panel we net out the influence of corresponding  $\mu$ . We first sort firms by  $\mu$  into 10 portfolios and then within each  $\mu$  decile sort firms into two portfolios by corresponding  $\sigma$ . We then average the one-period returns across all  $\mu$ -sorted portfolios to create returns of two portfolios with similar levels of  $\mu$  but different  $\sigma$ . In right panel we reverse this procedure, and first sort firms by  $\sigma$  into 10 portfolios, and then within each  $\sigma$  portfolio sort firms by  $\mu$  into two portfolios. We then average the one-period returns across all  $\sigma$ -sorted portfolios to create returns of two the one-period returns across all  $\sigma$ -sorted portfolios to create returns of two portfolios. We then average the one-period returns across all  $\sigma$ -sorted portfolios to create returns of two portfolios with similar levels of  $\sigma$  but different  $\mu$ . We report the differences across the two conditionally sorted portfolios in both left and right panels. The t-statistics are reported in parentheses and adjusted following Newey and West (1987) with a lag of 6 months.

		Controlling $\sigma$ Ran	g for $\mu$ k	С	ontrolling $\mu$ Rank	for $\sigma$
	Low	High	High-Low	Low	High	High-Low
$\sigma_{I/H,P}$						
Excess Return	1.02% (2.74)	0.80% (1.65)	-0.22% (-1.30)	0.89% (1.89)	0.94% (2.39)	$0.05\% \ (0.34)$
CAPM Alpha	0.19%	-0.24%	-0.42%***	-0.10%	0.05%	0.14%
	(1.20)	(-1.08)	(-2.61)	(-0.49)	(0.27)	(1.01)
FF3 Alpha	0.12%	-0.25%	-0.37%***	-0.12%	-0.02%	0.10%
	(1.04)	(-1.86)	(-4.30)	(-0.81)	(-0.13)	(0.79)
FFC4 Alpha	0.19%	-0.10%	-0.29%***	-0.01%	0.11%	0.12%
	(1.71)	(-0.77)	(-3.29)	(-0.10)	(0.88)	(0.93)
FFCP5 Alpha	0.20%	-0.09%	-0.29%***	0.01%	0.10%	0.09%
	(1.70)	(-0.67)	(-3.23)	(0.06)	(0.82)	(0.72)
$\sigma_{\Delta IV,P}$						
Excess Return	0.98%	0.84%	-0.14%	0.88%	0.94%	0.06%
	(2.61)	(1.76)	(-0.87)	(2.08)	(2.19)	(0.81)
CAPM Alpha	0.13%	-0.18%	-0.31%**	-0.05%	0.00%	0.05%
	(0.90)	(-0.82)	(-2.22)	(-0.32)	(0.01)	(0.63)
FF3 Alpha	0.06%	-0.20%	-0.26%***	-0.09%	-0.04%	0.05%
	(0.57)	(-1.37)	(-2.73)	(-0.85)	(-0.30)	(0.67)
FFC4 Alpha	0.15% (1.47)	-0.06% (-0.44)	-0.21%** (-2.14)	0.01% (0.12)	$0.08\% \ (0.59)$	0.07% (0.81)
FFCP5 Alpha	0.16%	-0.05%	-0.21%**	0.02%	0.09%	0.07%
	(1.43)	(-0.34)	(-2.03)	(0.18)	(0.64)	(0.85)

	Controlling for $\mu$ $\sigma$ Rank			Controlling for $\sigma$ $\mu$ Rank			
	Low	High	High-Low	Low	High	High-Low	
$\sigma_{IV,P}$							
Excess Return	1.00%	0.82%	-0.18%***	0.96%	0.86%	-0.10%	
	(2.31)	(1.97)	(-2.78)	(2.89)	(1.59)	(-0.35)	
CAPM Alpha	0.06%	-0.11%	-0.17%***	0.20%	-0.24%	-0.44%*	
	(0.34)	(-0.67)	(-2.67)	(1.46)	(-0.91)	(-1.75)	
FF3 Alpha	0.01%	-0.15%	-0.16%**	0.12%	-0.25%	-0.38%**	
	(0.10)	(-1.25)	(-2.56)	(1.18)	(-1.51)	(-2.57)	
FFC4 Alpha	0.12%	-0.03%	-0.15%**	0.17%	-0.08%	-0.24%	
	(0.99)	(-0.29)	(-2.56)	(1.66)	(-0.46)	(-1.58)	
FFCP5 Alpha	0.14%	-0.03%	-0.17%***	0.17%	-0.06%	-0.24%	
	(1.07)	(-0.27)	(-2.88)	(1.61)	(-0.36)	(-1.48)	

Table IX: Double-Sorted Portfolios by betas and  $\sigma$ s. For each  $\sigma$  measure in the left panel we net out the influence of beta. We first sort firms by beta into 10 portfolios and then within each beta decile sort firms into two portfolios by corresponding  $\sigma$ . We then average the one-period returns across all beta-sorted portfolios to create returns of two portfolios with similar levels of beta but different  $\sigma$ . In right panel we reverse this procedure, and first sort firms by  $\sigma$  into 10 portfolios, and then within each  $\sigma$  portfolio sort firms by beta into two portfolios. We then average the one-period returns across all  $\sigma$ -sorted portfolios to create returns of two portfolios with similar levels of  $\sigma$  but different beta. We report the differences across the two conditionally sorted portfolios in both left and right panels. The t-statistics are reported in parentheses and adjusted following Newey and West (1987) with a lag of 6 months.

	Controlling for Beta $\sigma$ Rank			Controlling for $\sigma$ Beta Rank			
	Low	High	High-Low	Low	High	High-Low	
$\sigma_{I/H,P}$							
Excess Return	1.04% (2.77) 0.20%	0.78% (1.62) -0.25%	-0.26% (-1.45) -0.45%***	0.91% (2.39) 0.05%	0.91% (1.92) -0.10%	0.00% (0.01) -0.16%	
CAPM Alpha	(1.26) 0.13%	(-1.13) -0.26%	(-2.66) -0.39%***	$(0.37) \\ 0.03\%$	(-0.47) -0.17%	(-1.17) -0.20%	
FF3 Alpha	$(1.09) \\ 0.19\%$	(-1.97) - $0.10\%$	(-4.74) -0.29%***	$(0.35) \\ 0.10\%$	(-1.00) - $0.01\%$	(-1.47) -0.11%	
FFC4 Alpha	(1.67) 0.21%	(-0.78) -0.10%	(-3.21) -0.31%***	$(0.96) \\ 0.10\%$	(-0.06) 0.01%	(-0.79) -0.09%	
FFCP5 Alpha	(1.72)	(-0.72)	(-3.31)	(0.92)	(0.06)	(-0.65)	
$\sigma_{\Delta IV,P}$							
Excess Return	$\begin{array}{c} 1.08\% \\ (2.83) \\ 0.22\% \end{array}$	$\begin{array}{c} 0.75\%\ (1.56)\ -0.27\%\end{array}$	-0.33%** (-1.98) -0.49%***	$0.91\% \ (2.37) \ 0.06\%$	0.91% (1.92) - $0.11\%$	-0.01% (-0.04) -0.16%	
CAPM Alpha	$(1.41) \\ 0.15\%$	(-1.23) - $0.28\%$	(-2.99) -0.43%***	$(0.40) \\ 0.04\%$	(-0.49) - $0.18\%$	(-1.16) -0.22%	
FF3 Alpha	$(1.20) \\ 0.23\%$	(-2.12) -0.14%	(-4.43) -0.38%***	$(0.45) \\ 0.10\%$	(-1.04) - $0.01\%$	(-1.56) -0.12%	
FFC4 Alpha	$(2.01) \\ 0.25\%$	(-1.08) -0.14%	(-3.51) -0.39%***	$(0.97) \\ 0.11\%$	(-0.08) -0.00%	(-0.80) -0.11%	
FFCP5 Alpha	(2.05)	(-1.01)	(-3.57)	(0.99)	(-0.00)	(-0.75)	

	Controlling for Beta $\sigma$ Rank			Controlling for $\sigma$ Beta Rank		
	Low	High	High-Low	Low	High	High-Low
$\sigma_{IV,P}$						
	1.04%	0.79%	-0.25%	0.91%	0.91%	0.00%
Excess Return	(2.76)	(1.63)	(-1.43)	(2.38)	(1.93)	(0.03)
	0.20%	-0.25%	$-0.45\%^{***}$	0.05%	-0.10%	-0.15%
CAPM Alpha	(1.25)	(-1.14)	(-2.68)	(0.35)	(-0.45)	(-1.11)
	0.12%	-0.26%	$-0.38\%^{***}$	0.03%	-0.17%	-0.20%
FF3 Alpha	(1.05)	(-1.90)	(-4.40)	(0.33)	(-1.00)	(-1.45)
	0.19%	-0.09%	-0.28%***	0.10%	-0.01%	-0.11%
FFC4 Alpha	(1.62)	(-0.72)	(-2.91)	(0.93)	(-0.05)	(-0.78)
	0.20%	-0.09%	-0.29%***	0.10%	0.01%	-0.10%
FFCP5 Alpha	(1.64)	(-0.64)	(-2.94)	(0.93)	(0.04)	(-0.68)

Table X: Fama-MacBeth Cross-Sectional Regressions. This table presents the firm-level cross sectional regressions of equity excess returns on  $\sigma$ s after controlling for the levels of the option-based variable  $\mu$ s, log market capitalization MV, log book-to-market ratio BM, Realized Volatility over past year RV, log at the money put option volume and open interest, VOL and OI, monthly stock volume VOLUME, and idiosyncratic volatility IDIOVOL. Model 1 presents cross-sectional regression results using  $\sigma_{I/H,P}$ . Model 2 presents results using  $\sigma_{\Delta IV,P}$ . Model 3 presents results using  $\sigma_{IV,P}$ . The coefficients and their Newey-West (1987) t-statistics are reported (t-statistics in parentheses). The last two rows report the  $R^2$  and Adjusted  $R^2$  values.

	1	2	3
INTERCEPT	1.830 ** (2.03)	2.075 ** (2.47)	1.820 * (1.97)
σ	-4.026*** (-3.18)	-9.343 *** (-6.25)	-4.585 *** (-3.41)
$\mu$	-1.935** (-2.54)	-2.626 (-1.15)	-1.754 ** (-2.17)
MV	-0.276** (-2.08)	-0.190 (-1.48)	-0.273 ** (-2.05)
BM	-0.078 (-0.87)	-0.053 (-0.58)	-0.080 (-0.89)
RV	-1.208 (-1.16)	-0.184 (-0.21)	$\begin{array}{c} 0.636 \\ (0.78) \end{array}$
$VOL_{P,ATM}$	-0.059 (-1.36)	-0.083* (-1.86)	-0.055 (-1.28)
$OI_{P,ATM}$	-0.077 (-1.46)	-0.092* (-1.74)	-0.075 (-1.43)
VOLUME	0.214 ** (2.43)	$\begin{array}{c} 0.136 \\ (1.55) \end{array}$	$\begin{array}{c} 0.211 \ ^{**} \\ (2.39) \end{array}$
IDIOVOL	-1.901 (-1.25)	-2.219 (-1.45)	-1.930 (-1.27)
$R^2$ Adj. $R^2$	$0.0846 \\ 0.0758$	0.0797 0.0708	$0.0848 \\ 0.0759$

Table XI: Fama-MacBeth Cross-Sectional Regressions. This table presents the firm-level cross sectional regressions of  $\sigma$ s on the proxies of heterogeneous beliefs. Panel A shows the results of univariate regressions. Model 1 Panel A presents the firm-level cross sectional regressions of  $\sigma$ s on the dispersion of analysts' forecast. Model 2 presents the firm-level cross sectional regressions of  $\sigma$ s on the dispersion of analysts' forecast. Model 2 presents the firm-level cross sectional regressions of  $\sigma$ s on idiosyncratic volatility. The coefficients and their Newey-West (1987) t-statistics are reported (t-statistics in parentheses). The last two rows report the  $R^2$  and Adjusted  $R^2$  values. Panel B presents the firm-level cross sectional regressions of  $\sigma$ s on the dispersion of the standard deviations of IV measures on the proxies of heterogeneous beliefs after controlling for option volumes, option open interests, leverage, liquidity (bid-ask spread) and beta. Model 1 presents the firm-level cross sectional regressions of  $\sigma$ s on idiosyncratic volatility. The coefficients and their Newey-West (1987) t-statistics are reported (t-statistics in parentheses). The last two rows report of  $\sigma$ s on idiosyncratic volatility. The coefficients and their Newey-West (1987) t-statistics are reported (t-statistics in parentheses). The last two rows report the  $R^2$  and Adjusted  $R^2$  values.

i and A. Onivariate Regression							
	$\sigma_{I/H,P}$		$\sigma_{\Delta I}$	IV,P	$\sigma_{IV,P}$		
	(1)	(2)	(3)	(4)	(5)	(6)	
INTERCEPT	$0.035^{***}$ (19.61)	$0.013^{***}$ (6.25)	$0.038^{***}$ (72.34)	$0.020^{***}$ (25.77)	$0.037^{***}$ (19.93)	$0.016^{***}$ (9.56)	
DISPERSION	$0.014^{***}$ (2.80)		$0.008^{***}$ (6.25)		$0.010^{***}$ (6.34)		
IDIOVOL		$0.204^{***}$ (10.63)		$0.160^{***}$ (25.44)		$0.197^{***}$ (23.37)	
$R^2$ $Adj.R^2$	$0.0492 \\ 0.0365$	$0.1292 \\ 0.1171$	$0.0337 \\ 0.0202$	$0.1404 \\ 0.1282$	$0.0455 \\ 0.0325$	$0.1346 \\ 0.1224$	

Panel A: Univariate Regression

		Panel B: M	ultivariate Re	egression		
	$\sigma_{I/}$	H,P	$\sigma_{\Delta I}$	IV,P	$\sigma_{IV,P}$	
	(1)	(2)	(3)	(4)	(5)	(6)
INTERCEPT	$\begin{array}{c} 0.021 \ ^{***} \\ (5.33) \end{array}$	-0.001 (-0.27)	0.043 *** (32.48)	0.027 *** (26.97)	0.023 *** (11.00)	$\begin{array}{c} 0.001 \\ (0.62) \end{array}$
DISPERSION	$\begin{array}{c} 0.011 & *** \\ (2.71) \end{array}$		0.006 *** (4.21)		0.008 *** (4.49)	
IDIOVOL		0.176 *** (17.77)		0.152 *** (30.62)		0.186 *** (20.52)
BASPREAD	$\begin{array}{c} 4.726 \ ^{***} \\ (4.35) \end{array}$	3.661 *** (4.19)	4.137 *** (4.21)	$\begin{array}{c} 2.411 & *** \\ (4.15) \end{array}$	$\begin{array}{c} 4.717 & *** \\ (4.25) \end{array}$	3.464 *** (4.09)
LEVERAGE	-0.007** (-2.36)	0.005 (1.27)	-0.006 *** (-3.81)	0.002 (1.43)	-0.009 *** (-3.77)	$0.002 \\ (0.87)$
BETA	0.004 *** (6.66)	0.001 * (1.77)	0.002 *** (6.53)	-0.000 (-0.64)	$\begin{array}{c} 0.003 & *** \\ (3.94) \end{array}$	-0.000 (-0.06)
$VOL_{P,ATM}$	0.004 *** (7.34)	0.003 *** (8.35)	-0.000 $(-1.57)$	-0.000 * (-1.73)	0.004 *** (10.09)	$\begin{array}{c} 0.004 & *** \\ (8.86) \end{array}$
$OI_{P,ATM}$	-0.001 *** (-3.78)	-0.000 (-0.11)	-0.002 *** (-6.83)	-0.001 *** (-5.56)	-0.001 *** (-3.63)	-0.000 (-0.89)
$R^2$ Adj.R <sup>2</sup>	$0.1794 \\ 0.1107$	$0.2368 \\ 0.1708$	$0.1714 \\ 0.0982$	$0.2392 \\ 0.1727$	$0.1858 \\ 0.1166$	$0.2513 \\ 0.1857$

Panel B: Multivariate Regression

cions of IV Measures: Panel Regressions. This table presents the panel regressions of $\sigma$ s on explanatory	liefs measure $\sigma$ s on the level variable $\mu$ s, $\sigma_{i,t} = a_0 + a_1 \mu_{i,t} + FE_t + \epsilon_{i,t}$ for the period from January 1996 to	l deviation of implied volatility measure for firm i at time t; $\mu_{i,t}$ is the corresponding mean of the option-	$E_t$ is the time fixed effect. Model 2 regresses $\sigma$ on its prior-month value, $\sigma_{i,t} = a_0 + a_2 \sigma_{i,t-1} + F E_t + \epsilon_{i,t}$ ,	ard deviation of IV measure for firm i at time t. Model 3 includes both the level $\mu$ , the prior value of	$a_3VOL_{P,i,t} + a_4OI_{P,i,t} + a_5MV_{i,t} + a_6BM_{i,t} + a_7SP500_t + a_9CON_t + a_{10}EXP_t + \epsilon_i, t$ , where $MV_{i,t}$ is	zation, $BM_{i,t}$ is book-to-market ratio, $SP500_{i,t}$ is value-weighted return on S&P 500 index (including	hm of corresponding volume and open interest of ATM puts, $CON_t$ is a dummy variable which is equal	to Nov 2001 or from Dec 2007 to Jun 2009, and equal to 0 otherwise, and $EXP_t$ is a dummy variable	trom Jan 1996 to Dec 1999, or from Jan 2005 to Jul 2007, and equal to 0 otherwise.	
Table XII: Determinants of the standard deviations of IV Measures: Panel Regressions.	variables. Model 1 regresses the dispersion of beliefs measure $\sigma s$ on the level variable $\mu s$ ,	August 2015. $\sigma_{i,t}$ is the corresponding standard deviation of implied volatility measure f	based predictive measure for firm $i$ at time $t$ ; $FE_t$ is the time fixed effect. Model 2 regre	where $\sigma_{i,t-1}$ is the corresponding lagged standard deviation of IV measure for firm i s	$\sigma$ , and controls: $\sigma_{i,t} = a_0 + a_1\mu_{i,t} + a_2\sigma_{i,t-1} + a_3VOL_{P,i,t} + a_4OI_{P,i,t} + a_5MV_{i,t} + a_6B$	natural logarithm of the firm's market capitalization, $BM_{i,t}$ is book-to-market ratio, $\Sigma$	dividend), $VOL_{P,i,t}$ , $OI_{P,i,t}$ are natural logarithm of corresponding volume and open ir	to 1 if the date of observation is from Mar 2001 to Nov 2001 or from Dec 2007 to Jun $2$	which is equal to 1 if the date of observation is from Jan 1996 to Dec 1999, or from Ja	

	(3)	0.1017 *** (0.0015)	** 0.1380 *** (0.0061)	$\begin{array}{c} 0.4161 & *** \\ (0.0135) \end{array}$	-0.0498 *** (0.0111)	-0.0203 $(0.0143)$	0.0183 (0.0164)	-2.7412 *** (0.2143)	-0.1637 *** (0.0363)	-0.0198 (0.0284)	No	0.3914
IV, P	(2)	×	$0.3869 \ ^{*}$ (0.0074)								Yes	0.2476
(c)	(1)	$0.1268 ^{**:}$ (0.0017)									Yes	0.4252
	Variable	$\mu_{IV,P}$	$\sigma_{IV,P,t-1}$	$VOL_{P,ATM}$	$OI_{P,ATM}$	MV	BM	SP500	CON	EXP	Time FE	$R^2$
	(3)	-0.0636 $(0.0513)$	$\begin{array}{c} 0.2800 & *** \\ (0.0122) \end{array}$	0.2199 *** (0.0109)	-0.0258 ** (0.0107)	-0.7003 *** (0.0172)	-0.2484 *** (0.0162)	0.0883 (0.1385)	-0.2509 *** (0.0205)	-0.3144 *** (0.0203)	No	0.2334
JV, P	(2)		0.4013 *** (0.0120)								Yes	0.1820
(b) <u></u>	(1)	-0.1174 ** ( 0.0543)									Yes	0.02873
	Variable	$\mu \Delta IV, P$	$\sigma \Delta IV, P, t-1$	$VOL_{P,ATM}$	$OI_{P,ATM}$	MV	BM	SP500	CON	EXP	Time FE	$R^2$
	(3)	0.0779 *** (0.0043)	0.2457 *** (0.0091)	$\begin{array}{c} 0.6461 \ ^{***} \\ (0.0155) \end{array}$	-0.0488 *** (0.0114)	-0.8591 *** (0.0173)	-0.3215 *** (0.0199)	-1.1592 *** (0.2746)	0.7672 *** (0.0352)	-1.0543 *** (0.0281)	No	0.2792
/H,P	(2)		$\begin{array}{c} 0.3441 \ ^{***} \\ (0.0131) \end{array}$								Yes	0.2101
(a) I,	(1)	$\begin{array}{c} 0.0916 & *** \\ (0.0071) \end{array}$									Yes	0.1671
	Variable	$\mu_I/H,P$	$\sigma_{I/H,P,t-1}$	$VOL_{P,ATM}$	$OI_{P,ATM}$	MV	BM	SP500	CON	EXP	Time FE	$R^2$

Table XIII: Difference-in-difference estimation using Regulation SHO enacted in 2005. Panel A presents the difference-in-difference results for a short-sale constraint shock on  $\sigma$ s. Panel B presents the difference-indifference results for a short-sale constraint shock on the predictability of underlying stock returns from  $\sigma$ s. EFF is a dummy variable which equals one from January 2005 to August 2007 and zero otherwise. SHO is a dummy variable which equals one if the stock is a pilot stock and zero otherwise.

Panel A: Short-sale constraints on $\sigma$							
	(1)	(2)	(3)				
INTERCEPT	$\frac{12.6942^{***}}{(0.3146)}$	$11.2206^{***} \\ (0.3200)$	-1.5315 *** (0.2575)				
$\mu$	0.0712 *** (0.0072)	-0.1612 ** (0.0816)	$\begin{array}{c} 0.0971 \ ^{***} \\ (0.0017) \end{array}$				
$\sigma_{t-1}$	0.2392 *** (0.0144)	0.2717 *** (0.0170)	0.1182 *** (0.0066)				
SHO X EFF	$0.1600 \ ^{**}$ (0.0689)	$0.0608 \\ (0.0449)$	$0.1196^{*}$ (0.0674)				
$\mathbf{EFF}$	-1.1979 *** (0.0366)	-0.1235 *** (0.0261)	$\begin{array}{c} 0.1424 \ ^{***} \\ (0.0391) \end{array}$				
SHO	-0.0240 (0.0453)	$0.0159 \\ (0.0496)$	-0.0063 (0.0453)				
MV	-0.8123 *** (0.0207)	-0.6001 *** (0.0204)	-0.0329 ** (0.0152)				
BM	-0.2681 *** (0.0266)	-0.2098 *** (0.0216)	0.0288 (0.0210)				
$VOL_{P,ATM}$	0.6256 *** (0.0175)	0.2018 *** (0.0117)	$\begin{array}{c} 0.3971 \ ^{***} \\ (0.0146) \end{array}$				
$OI_{P,ATM}$	-0.0247 * (0.0129)	-0.0522 *** (0.0118)	-0.0633 *** (0.0122)				
SP500	-2.1474 *** (0.4060)	$\begin{array}{c} 0.3636 & *** \\ (0.1321) \end{array}$	-2.3925 *** (0.2332)				
$R^2$	0.2491	0.2315	0.3826				

	(1)	(2)	(3)
INTERCEPT	3.1138 *** (0.4217)	3.3211 *** (0.4361)	2.9503 *** (0.4250)
σ	-14.164 *** (2.1694)	-8.0208 *** (2.1070)	-19.297 *** (1.9891)
$\mu$	-0.6268 (0.5310)	-0.5332 (2.9900)	$0.5023 \\ (0.5704)$
$\sigma$ X SHO X EFF	-7.0023 (6.5071)	$7.3694 \\ (5.0468)$	-5.3849 (6.4401)
$\sigma$ X EFF	$12.3485^{***}$ (4.4063)	-0.9803 (3.7874)	$14.2871^{***}$ (4.4356)
$\sigma$ X SHO	5.0962 * (2.9923)	-0.1441 (2.9640)	6.6383 ** (2.9350)
SHO X EFF	$0.2356 \\ (0.2415)$	-0.3403 (0.2536)	$0.2095 \\ (0.2496)$
EFF	-0.1713 (0.1496)	0.3676 ** (0.1724)	-0.2621* (0.1555)
SHO	-0.1799 (0.1299)	$\begin{array}{c} 0.0311 \\ (0.1313) \end{array}$	$-0.2555^{*}$ (0.1322)
MV	-0.1618 *** (0.0480)	-0.0890 * (0.0480)	-0.1602 *** (0.0481)
BM	0.1896 *** (0.0530)	0.2034 *** (0.0524)	$0.1848 ^{***}$ (0.0531)
RV	1.9444 *** (0.3582)	1.6298 *** (0.2682)	2.0301 *** (0.4346)
$VOL_{P,ATM}$	0.0695 * (0.0401)	0.0280 (0.0398)	0.0856 ** (0.0401)
$OI_{P,ATM}$	-0.1275 *** (0.0342)	-0.1220 *** (0.0341)	-0.1296 *** (0.0342)
VOLUME	-0.0144 (0.0529)	-0.0832 (0.0524)	-0.0069 (0.0530)
IDIOVOL	-4.6067 *** (1.0838)	-3.7053 *** (1.0752)	-4.8481 *** (1.0902)
$R^2$	0.2201	0.1309	0.2649

Panel B: Short-sale constraints on  $\sigma$  return predictability

Figure 1: Time Series of Medians of Standard Deviations of Option-based Belief Measures. This figure presents the cross sectional medians of the standard deviations of the implied volatility measures across all stocks in our sample for each month during the sample period from Jan 1996 to Aug 2015. Dashed gray areas are contraction periods; shaded gray areas are expansion periods.

